Discrete Green's function and maximum principles

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Abstract

We consider linear second-order elliptic equation with Dirichlet boundary conditions. We discretize this problem by means of the *hp* finite element method. In this context, the discrete Green's function G_{hp} is naturally defined. It can be used to express the finite element solution u_{hp} in the form of integral operator:

$$u_{hp}(z) = \int_\Omega G_{hp}(x,z) f(x) \, \mathrm{d}x,$$

where f stands for the right-hand side. From this expression follows that the discrete maximum principle is satisfied if and only if $G_{hp} \geq 0$ in the domain Ω.

An explicit expression of the discrete Green's function can be used to prove the discrete maximum principle in 1D under the condition that all the elements are shorter than a critical length. This critical length depends on the polynomial degree p prescribed on the element. It can be calculated explicitely for $p \leq 4$ and numerically for $p \geq 5$. From these results we conjecture [1] that the critical length divided by the length of the whole domain is bounded from below by 9/10.

In the talk we give a survey of properties of the discrete Green's function with the aim to show how to generalized the above mentioned result to more general equations and/or to higher spatial dimension. The results about the discrete Green's function might be of independent interest.

References

[1] T. Vejchodský, P. Šolín: Discrete maximum principle for higher-order finite elements in 1D, submitted, 2006.