Fictitious domain approach for the numerical realization of PDEs with stochastic data and geometry

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PDE solvers which are based on the fictitious domain (FD) formulations represent nowadays one of efficient tools for solving large scale algebraic systems arising from their discretizations. The main reason for their popularity is that they allow to transform the original problem defined in a domain D with a possible complicated geometry to a new one solved in a simple shaped domain $\hat{D}$ (a box, e.g.) which is called fictitious and contains $\overline{D}$. In $\hat{D}$ one can use fairly structured meshes which enable us to use easily very efficient solvers such as domain decomposition and multigrid type methods, special fast Fourier and cyclic reduction algorithms and special preconditioning techniques.

FD solvers have additional advantages when used in problems with moving boundary, e.g. shape optimization, free boundary problems and problems with stochastic geometry which are the focus of this paper. To see it let us suppose that a linear boundary value problem is solved by a standard finite element method. Then the following steps have to be performed after every change of the shape: (i) *remeshing the new configuration*; (ii) *assembling the new stiffness matrix and the right-hand side of the linear algebraic system*; (iii) *solving this new system*. Thus the efficiency of solving the discrete problems is crucial. Hereafter, we will explore a fictitious domain method with nonfitted meshes as a possible way to increase efficiency: indeed, this approach avoids completely step (i) and partially step (ii) since the stiffness matrix remains the same for every admissible domain.

There are many ways how to associate the new problem in $\hat{D}$ with the original one defined in D. We can use a penalty method, Lagrange multipliers or optimal control approach.

In [1], we present a method for the numerical realization of PDEs with stochastic data and geometry. This method is based on the suitable combination of fictitious domain approach based on boundary Lagrange multipliers defined on the boundary and polynomial chaos [2]. The fundamental step of a stochastic PDE approach is adding one or more stochastic variables to the deterministic PDE, as an alternative to sampling techniques. Then all stochastic data in PDE are decomposed using fully orthonormal polynomial basis into separable deterministic and stochastic components. For spatial discretization we use a fictitious domain method based on the boundary Lagrange multipliers with the standard mixed finite elements. For more details and model examples we refer to [1].

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