On a traffic problem

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A massive traffic is the phenomenon of our civilization. We will consider a class of *microscopic follow-the-leader models*, see [1]:

$$x_i' = y_i \,, \tag{1}$$

$$y_i' = V(x_{i+1} - x_i) - y_i \,, \quad x_{N+1} = x_1 + L \tag{2}$$

$i = 1, \ldots, N$. The system (1)&(2) models N cars on a circular road of the length L. The difference $h_i \equiv x_{i+1} - x_i$ is called *headway* (of the i-th car). The function $V = V(x_{i+1} - x_i) = V(h_i)$ in (2) is *optimal velocity function.* In particular, we will consider $r \mapsto V(r)$,

$$V(r) \equiv V^{max} \frac{\tanh\left(a(r-1)\right) + \tanh(a)}{1 + \tanh(a)} \,,$$

where V^{max} and a are positive constants.

There exists a family of *quasi-stationary solutions*. In certain parameter regions the quasi-stationary solutions co-exist with periodic solutions, see [1]. The stability exchange is due to a Hopf bifurcation. It was observed in [1] that periodic solutions often become *unphysical* namely, a headway become negative.

Let us analyze the case that since certain time tE a trajectory of (1)&(2) is unphysical. At this time tE a car is about to **overtake** the foregoing one. Our objective is to develop a reasonable model of the overtaking. The resulting model is a piecewise smooth dynamical system composed by pieces of (1)&(2).

We investigate asymptotic properties of the overtaking model as $t \to \infty$. We will report on invariant objects we observed. These objects exhibit spatial-temporal symmetries.

References

[1] I. Gasser, G. Sirito, B. Werner: *Bifurcation analysis of a class of 'car following' traffic models*. Physica D 197 (2004), pp. 222–241.