Finite volume WLSQR scheme and its applications for transonic flows

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This contribution deals with the development of weighted least square (WLSQR) reconstruction of cell-averaged data. The WLSQR reconstruction is then used as a key part of a high order finite volume scheme for simulation of transonic flows.

The WLSQR interpolation is in some sense similar to the so-called ENO (*essentially non-oscillatory*) method. The standard ENO reconstruction is obtained in the following steps:

1. forming several sets of cells (so-called stencils),
2. computing an interpolation polynomial for each stencil, and
3. selecting a polynomial with smallest variation.

The main difficulty with this algorithm is the creation of admissible stencils (i.e. stencils which give a unique solution in step 2 of the algorithm).

On the other hand, the WLSQR method proceeds in this way:

1. forming one fixed stencil with more cells than unknown coefficients of the polynomial, and then
2. solving an overdetermined system of equation in the least square sense.

In order to mimic ENO or weighted ENO schemes we introduce data dependent weights into the system in step 2. The weights are chosen in such a way, that they are small for discontinuous data and big for smooth data.

We analyse the one-dimensional WLSQR algorithm for the case of smooth data and a discontinuous data $u(x) = \theta(x)$ (i.e. 0 for $x < 0$ and 1 for $x \geq 0$) and we show in both cases estimates of total variation of the interpolant.

Next we show several numerical results obtained with an semi-implicit finite volume method with WLSQR reconstruction:

- inviscid flow in a 2D test channel,
- inviscid flow through 2D turbine cascade using triangular meshes,
- turbulent flows through turbine cascades, and
- inviscid 3D flows in turbines and channels.

The comparison of numerical results with experimental data or with results obtained by other researchers demonstrate the applicability of the WLSQR method.

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