An efficient implementation of the semi-implicit discontinuous Galerkin method for compressible flow simulation

Vít Dolejší

Charles University Prague, Faculty of Mathematics and Physics, Czech Republic
e-mail: dolejsi@karlin.mff.cuni.cz

ABSTRACT

Our aim is to developed a sufficiently robust, accurate and efficient numerical scheme for the simulation of compressible flow, which is described by the system of the Euler (inviscid model) and/or Navier-Stokes (viscous model) equations. Among several types of numerical schemes the discontinuous Galerkin method (DGM) seems to be a promising technique for the discretization of the Euler and/or Navier-Stokes equations. DGM is based on a discontinuous piecewise-polynomials approximations and its main advantages are the discontinuous approximation (important for transonic flow regimes), high order of approximation, local character of the method and easy parallelization. For survey about DGM see e.g., [CKS00].

The application of DGM for compressible flow simulation was treated in many papers, see e.g., [BR97], [BO99], [HH02], [VV02] and also our research, e.g., [DF04], [Dol04]. Although authors mostly claim that DGM is very suitable numerical scheme for the compressible flow simulation they admit one disadvantage of DGM: a high computational cost which prevents DGM from practical application. Therefore efficient algoritmization and implementation exhibit a challenging task.

Within this contribution we present a semi-implicit numerical scheme which is based on DGM for the space discretization and the backward difference formula for the time discretization. We pay the attention to an efficient implementation of this approach with respect to the CPU time. We discuss the choice of basis functions, numerical quadratures for integrations, matrix solver, adaptive choice of the time step and data representation. We present several comparisons of different choices and propose the "best strategy" (from our opinion).

REFERENCES

[BR97] F. Bassi, S. Rebay: A high-order accurate discontinuous finite element method for the numerical solution of the compressible Navier–Stokes equations, J. Comput. Phys. 131:267-279, 1997.

[BO99] C. E. Baumann, J. T. Oden: A discontinuous hp finite element method for the Euler and Navier-Stokes equations, Int. J. Numer. Methods Fluids 31:79-95, 1999.

[CKS00] B. Cockburn and G. E. Karniadakis and C.–W. Shu (eds.): Discontinuous Galerkin Methods, Lecture Notes in Computational Science and Engineering 11., Springer, Berlin, 2000.

[DF04] V. Dolejší, M. Feistauer: Semi-implicit discontinuous Galerkin finite element method for the numerical solution of inviscid compressible flow, J. Comput. Phys. 198(2):727-746, 2004.

[Dol04] V. Dolejší: On the discontinuous Galerkin method for the numerical solution of the Navier–Stokes equations, Int. J. Numer. Methods Fluids, 45:1083–1106, 2004.

[HH02] R. Hartmann, P. Houston: Adaptive discontinuous Galerkin finite element methods for the compressible Euler equations, J. Comput. Phys. 183:508-532, 2002.

[VV02] J. J. W. van der Vegt, H. van der Ven: Space-time discontinuous Galerkin finite element method with dynamic grid motion for inviscid compressible flows. I: General formulation, J. Comput. Phys. 182:546-585, 2002.