Scalable Algorithms for Contact Problems with Geometrical and Material Nonlinearities

Vít Vondrák, Zdeněk Dostál, Jiří Dobiáš, Svatopluk Pták

In 1991 Ch. Farhat and F.-X. Roux came up with a novel domain decomposition method called FETI (Finite Element Tearing and Interconnecting method) [1]. Its key ingredient is decomposition of the spatial domain into non-overlapping subdomains that are "glued" by Lagrange multipliers, so that, after eliminating the primal variables, the original problem is reduced to a small, relatively well conditioned, typically equality constrained quadratic programming problem that is solved iteratively.

If the FETI procedure is applied to an elliptic variational inequality, the resulting quadratic programming problem has not only the equality constraints, but also the non-negativity constraints. Even though the latter is a considerable complication as compared with linear problems, it seems that the FETI procedure should be even more powerful for the solution of variational inequalities than for the linear problems. The reason is that FETI not only reduces the original problem to a smaller and better conditioned one, but it also replaces for free all the inequalities by the bound constraints.

Such a formulation turned out to be a convenient starting point for development of efficient algorithms for variational inequalities that describe the equilibrium of a system of elastic bodies in mutual contact and several theoretically supported scalable algorithm for frictionless contact problems with linear elastic bodies were developed [2, 3, 4].

Herein we extend these results to problems with the geometric and material nonlinearities. The scalable algorithms mentioned above are combined with the nested iteration strategy, where the outer loop is concerned with the material and geometric nonlinear effects, contact geometry update, and equilibrium iterations.

Numerical experiments were carried out with our in-house general purpose finite element package PMD (Package for Machine Design) [5]. Our algorithms were tested on the class of Hertzian contact problems and a Persson's problem. Then the analytical solutions are known for both cases and they are used as benchmarks for evaluation of accuracy of our algorithms, and for comparison with numerical solutions taking into account both geometric and material non-linearities.

References

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