

**Hierarchical FEM:
Strengthened CBS Inequalities and Iterative Solvers**

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The finite element analysis is often done on a finite element mesh, which is constructed as multiple refinement of a given coarse initial mesh. In this case, the (geometric) hierarchy of meshes and finite element spaces can be used for development of multilevel iterative methods or preconditioners for the solution of corresponding algebraic finite element systems. These solvers can be constructed on the basis of hierarchical decomposition of finite element spaces into coarser finite element space and its complement. The angle of these subspaces or in other words the strengthened CBS inequality constant is an important characteristic playing role in convergence analysis of iterative methods as well as in efficiency analysis of some type of a posteriori error estimators.

The aim of the contribution is the following. First, it provides an overview of the estimates of the strengthened CBS inequality constant for hierarchical finite element spaces including some recent results. A special attention will be given to the robustness of these estimates to anisotropy, element shapes and jumps of the coefficients. Second, it shows the role of these estimates in convergence analysis of multilevel preconditioners and iterative solution methods for solving problems on both globally and locally refined grids. Finally, we indicate some possible more algebraic extensions of the hierarchical decomposition technique. Such extension can be also applied in the case of nonconforming finite elements.

Related papers:

O. Axelsson, R. Blaheta, Two simple derivations of universal bounds for the C.B.S. inequality constant, *Appl. Math.* 49(2004)

R. Blaheta, Nested tetrahedral grids and strengthened C.B.S. inequality, *Numerical Linear Algebra with Applications*, 10(2003), Issue 7, pp. 619-637

R. Blaheta, M. Neytcheva, S. Margenov: Uniform estimate of the constant in the strengthened CBS inequality for anisotropic non-conforming FEM systems, *Numerical Linear Algebra with Applications* 11(2004), issue 4, 309-326

R. Blaheta, M. Neytcheva, S. Margenov, Robust optimal multilevel preconditioners for nonconforming FE systems. *Numerical Linear Algebra with Applications*, 12(2005), pp. 495-514