An algorithm for solving nonsymmetric saddle-point linear systems arising in FDM

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The contribution deals with saddle-point linear systems arising in the fictitious domain method for the elliptic boundary value problems with mixed boundary conditions [2]. We shall propose a fast method for finding a pair $(\mathbf{u}, \boldsymbol{\lambda}) \in \mathbb{R}^n \times \mathbb{R}^m$ so that

$$\begin{pmatrix} \mathbf{A} & \mathbf{B}^\top \\ \mathbf{C} & \mathbf{0} \end{pmatrix} \begin{pmatrix} \mathbf{u} \\ \boldsymbol{\lambda} \end{pmatrix} = \begin{pmatrix} \mathbf{f} \\ \mathbf{g} \end{pmatrix}, \tag{1}$$

where the $n \times n$ matrix $\mathbf{A}$ is symmetric positive semi-definite, the $m \times n$ matrices $\mathbf{B}$ and $\mathbf{C}$ have full row-rank and the vectors $\mathbf{f}$, $\mathbf{g}$ are of the order n, m, respectively. We shall be interested especially in systems (1) with n large, $\mathbf{A}$ singular, $\mathbf{B}$, $\mathbf{C}$ sparse and m much smaller than n. Moreover we shall assume that the defect of $\mathbf{A}$, i.e. $l = n - rank\mathbf{A}$, is much smaller than m.

In order to treate the possible singularity of $\mathbf{A}$, we shall use the orthogonal projectors. This idea has been used in the FETI based domain decomposition methods [1], where the saddle-point linear systems are typically symmetric, i.e. $\mathbf{B} = \mathbf{C}$. We shall present an extension of the orthogonal projectors to the non-symmetric case in which $\mathbf{B} \neq \mathbf{C}$.

The FETI algorithms require to evaluate actions of a generalized inverse $\mathbf{A}^\dagger$. We shall show that, in the the context of the fictitious domain method, $\mathbf{A}^\dagger$ can be easily realized by the Moore-Penrose pseudoinverse and, moreover, its action can be computed using the highly efficient fast Fourier transform [3].

References

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[3] R. K. (2005): *Complexity of an algorithm for solving saddle-point systems with singular blocks arising in wavelet-Galerkin discretizations*, Appl. Math. **50**, 291–308.