High–order approximations of gradients of smooth functions in vertices of sharply regular triangulations

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Abstract

For a regular triangulation $\mathcal{T}_h$ without obtuse angles of a bounded open polygon $\Omega \subset \Re^2$, let $\mathcal{L}_h$ be the space of continuous functions, linear on the triangles from $\mathcal{T}_h$ and Π_h the operator of interpolation in the vertices of $\mathcal{T}_h$ from $C(\overline{\Omega})$ to $\mathcal{L}_h$. We characterize such vertices a of the triangulation $\mathcal{T}_h$ that there exist vectors f with the property that the linear combinations $f_1 \nabla \Pi_h(u)|_{T_1} + \ldots + f_n \nabla \Pi_h(u)|_{T_n}$ of the gradients $\nabla \Pi_h(u)|_{T_i}$ on the triangles $T_1, \ldots, T_n \in \mathcal{T}_h$ meeting a approximate the gradient $\nabla u(a)$ with an error of the size $O(h^2)$ for every sufficiently smooth function u.