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# PARALLEL PROGRAMMING AND OPTIMIZATION OF HEAT RADIATION INTENSITY

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#### Abstract

This article focuses on the practical possibilities of a suitable use of parallel programming during the computational processing of heat radiation intensity optimization across the surface of an aluminium or nickel mould. In practice, an aluminium or nickel mould is first preheated by infrared heaters located above the outer mould surface. Then the inner mould surface is sprinkled with a special PVC powder and the outer mould surface is continually warmed by infrared heaters. This is an energyefficient way to produce artificial leathers in the car industry (e.g., the artificial leather on a car dashboard). It is necessary to optimize the location of the heaters to approximately ensure the same heat radiation intensity across the whole outer mould surface during the warming of the mould (to obtain a uniform material structure and color tone of the artificial leather). The problem of optimization is complicated (moulds used in production are often very rugged, during the process of optimization we avoid possible collisions of two heaters as well as a heater and the mould surface). Using of gradient methods is not suitable for solving the problem (minimized function contains many local extremes). A genetic algorithm is used to optimize the location of the heaters. The optimization computation procedure is demanding in terms of the number of numerical operations (especially when the mould volume is large and the number of used infrared heaters is higher). In this article practical results of parallel programming during the calculation process of the evaluation function of every created individual (one possible solution of optimization problem using genetic algorithm) to define its fitness are given. The numerical calculations were performed by a Matlab code written by the authors. Numerical experiments are focused exclusively on the opportunities to use parallel programming to accelerate the optimization procedure.

#### 1. Introduction

This article focuses on the possibilities of parallel programming to accelerate computational optimization of heat radiation intensity on a mould surface. Our minimization problem has many local extremes. Using of gradient methods for finding

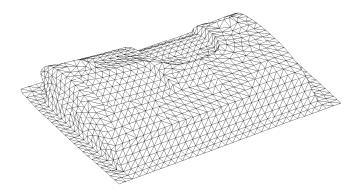


Figure 1: An aluminium mould of a passenger car dashboard part.

global minimum is therefore unsuitable, that is the reason why a genetic algorithm is used.

In practice, an aluminium or nickel mould is preheated by infrared heaters located above the outer mould surface. It is necessary to ensure the same heat radiation intensity (within a given tolerance) on the whole mould surface by finding suitable locations of the heaters. In this way the same material structure and colour of the artificial leather are assured. Moulds of different proportions (often very complicated) and with weight of approximately 300 kilograms are used in production (see Figure 1). The infrared heaters have a tubular form and their length is about 20 centimeters. Every heater is equipped with a mirror located above the radiation tube, which reflects heat radiation in a set direction.

## 2. The model of heat radiation on the mould surface

In this chapter a simplified mathematical model of heat radiation produced by infrared heaters on the outer mould surface is described. The heaters and the heated mould are represented in 3-dimensional Euclidean space  $E_3$  using the Cartesian coordinate system  $(O, x_1, x_2, x_3)$  with basis vectors  $e_1 = (1, 0, 0)$ ,  $e_2 = (0, 1, 0)$ ,  $e_3 = (0, 0, 1)$ .

Representation of a heater. A heater is represented by abscissa of length d (see Figure 2). The location of a heater is defined by the following parameters: (i) the coordinates of the heater centre  $S = [s_1, s_2, s_3]$ , (ii) the unit vector  $u = (u_1, u_2, u_3)$  of the heat radiation direction, where component  $u_3 < 0$  (i.e., the heater radiates "downward"), (iii) the vector of the heater axis  $r = (r_1, r_2, r_3)$ . Another way to determine the vector r is by using only the angle  $\varphi$  between the vertical projection of vector r onto the  $x_1x_2$ -plane and the positive part of axis  $x_1$  (the vectors u and r are orthogonal,  $0 \le \varphi < \pi$ ). The location of every heater Z can be defined by the following 6 parameters

$$Z: (s_1, s_2, s_3, u_1, u_2, \varphi). \tag{1}$$

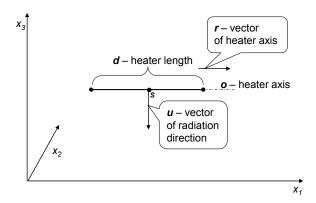


Figure 2: Schematic representation of the heater.

Representation of a mould. The outer mould surface P is described by elementary surfaces  $p_j$ , where  $j \in \{1, 2, ..., N\}$ . We have that  $P = \cup p_j$ , where  $j \in \{1, 2, ..., N\}$  and int  $p_i \cap$  int  $p_j = \emptyset$  for  $i \neq j$ ,  $1 \leq i, j \leq N$ . Every elementary surface  $p_j$  is described by the following parameters: (i) its centre of gravity  $T_j = [t_1^j, t_2^j, t_3^j]$ , (ii) the unit outer normal vector  $v_j = (v_1^j, v_2^j, v_3^j)$  at the point  $T_j$  (we suppose  $v_j$  faces "upwards" and therefore is defined through the first two components  $v_1^j$  and  $v_2^j$ ), (iii) the area  $c_j$  of the elementary surface. Every elementary surface  $p_j$  thus can be defined by the following 6 parameters

$$p_j: (t_1^j, t_2^j, t_3^j, v_1^j, v_2^j, c_j). (2)$$

Experimental measurement of heater radiation intensity. We need to know the heat radiation intensity in the heater surroundings to calculate the total radiation intensity on the outer mould surface. The heater manufacturer has not provided the distribution function of the heat radiation intensity in the heater surroundings. We set up the experimental measurement of the heat radiation intensity as follows. The location of the heater was Z:(0,0,0,0,0) in accordance with relation (1), i.e., the centre S of the heater lay at the origin of the Cartesian coordinate system  $(0, x_1, x_2, x_3)$ ; the unit radiation vector had coordinates u = (0, 0, -1) and the vector of the heater axis had coordinates r = (1,0,0). We assume the heat radiation intensity across the elementary surface  $p_i$  is the same as at the centre of gravity  $T_i$ . The heat radiation intensity at  $T_i$  depends on the position of this point (determined by the first three parameters in the elementary surface  $p_i$  given by relation (2)) and on the direction of the outer normal vector  $v_j$  at the point  $T_j$  (determined by the fourth and fifth parameters in the elementary surface  $p_i$  given by (2)). The heat radiation intensity I in the surroundings and below the heater was experimentally measured by a sensor at selected points  $a = [a_1, a_2, a_3, a_4, a_5]$  (the first three parameters describe the position of the centre of gravity of fictitious elementary surface and fourth and fifth parameters describe the direction of the outer normal vector in the point  $[a_1, a_2, a_3]$ ). We can use measured values I(a) of heat radiation intensity at the selected points a and linear interpolation function of five variables to calculate the heat radiation intensity I(b) for the general point  $b = [b_1, b_2, b_3, b_4, b_5]$  in the heater surroundings. Interpolation formula is described in details in [2], p. 148.

The general case of a heater location. For a heater in general position, we briefly describe the transformation of the previous Cartesian coordinate system  $(O, e_1, e_2, e_3)$  into a positively oriented Cartesian system (S, r, n, -u), where S is the centre of the heater, r is the heater axis vector, and u is the direction vector of the heat radiation. The vector n is determined by the cross product of the vectors -u and r (see more detail in [6], p. 6) and is defined by the following relation

$$n = (-u) \times r = \left( - \left| \begin{array}{cc} u_2 & u_3 \\ r_2 & r_3 \end{array} \right|, \left| \begin{array}{cc} u_1 & u_3 \\ r_1 & r_3 \end{array} \right|, - \left| \begin{array}{cc} u_1 & u_2 \\ r_1 & r_2 \end{array} \right| \right).$$

The vectors r, u and n are normalized to have unit length. Then we can define an orthonormal transformation matrix

$$\mathbf{A} = \begin{pmatrix} r_1 & n_1 & -u_1 \\ r_2 & n_2 & -u_2 \\ r_3 & n_3 & -u_3 \end{pmatrix}.$$

Let us recall that for the elementary surface  $p_j$ , the respective triples  $T_j$  and  $v_j$  represent its centre of gravity and its outer normal vector in the Cartesian coordinate system  $(O, e_1, e_2, e_3)$ . If S is the trio representing (in  $(O, e_1, e_2, e_3)$ ) the centre of the heater that determines the coordinate system (S, r, n, -u), then  $T_j$  and  $v_j$  are transformed as follows

$$(T_j')^T = \mathbf{A}^T (T_j - S)^T \quad \text{and} \quad (v_j')^T = \mathbf{A}^T v_j^T,$$
 (3)

where  $T_j^{'}$  and  $v_j^{'}$  are the coordinates in (S, r, n, -u). In this way, we transform the general case of heater location to the measured case and we can calculate heat radiation intensity by using linear interpolation described in previous paragraph "Experimental measurement of heater radiation intensity" (transformed point  $T_j^{'}$  and vector  $v_j^{'}$  correspond to the point b in previous paragraph).

Calculation of total heat radiation intensity. Now we describe the numerical computation procedure for the total heat radiation intensity on the mould surface. We denote by  $L_j$  the set of all heaters radiating on the jth elementary surface  $p_j$   $(1 \le j \le N)$  for the fixed locations of heaters, and  $I_{jl}$  the heat radiation intensity of the lth heater on the  $p_j$  elementary surface. Then the total radiation intensity  $I_j$  on the elementary surface  $p_j$  is given by the following relation

$$I_j = \sum_{l \in L_j} I_{jl} \ . \tag{4}$$

The producer of artificial leathers recommends a constant value of heat radiation intensity across the whole outer mould surface. Let us denote this constant value as  $I_{rec}$ . We can define F, the aberration of the heat radiation intensity, by the relation

$$F = \frac{\sum_{j=1}^{N} |I_j - I_{rec}| c_j}{\sum_{j=1}^{N} c_j}$$
 (5)

and the aberration  $\tilde{F}$  by the relation

$$\tilde{F} = \sqrt{\sum_{j=1}^{N} (I_j - I_{rec})^2 c_j} . {(6)}$$

We highlight that  $c_j$  denotes the area of the elementary surface  $p_j$ . We need to find the location of heaters such that value of aberration F (alternatively aberration  $\tilde{F}$ ) will be within specified tolerance.

## 3. The optimization of the location of the heaters

Function F defined by (5) has many local extremes. Using gradient methods for finding minimum of the function F is not appropriate. If we use a gradient method, there is a high likelihood that we find only local minimum of function F. Therefore, we use a genetic algorithm for finding global minimum of function F(i.e., to optimization of the locations of the heaters). A disadvantage of genetic algorithms is its computational demand and slow convergence. A genetic algorithm is described in more details in [1] and [3]. Implementation of this algorithm for solution of our optimization problem is described in details in [4]. The location of every heater is defined in accordance with the relation (1) by 6 parameters. Therefore, 6M parameters are necessary to define the locations of all M heaters. One individual in genetic algorithm represents one possible location of the all 6M heaters. In the algorithm we successively construct populations of individuals. Every population includes Q individuals where every individual is a potential solution of our problem (in contrast with the gradient methods, where only one potential solution in each iteration exists). We use operators one-point crossover (operator that combines two individual to produce a new individual) and mutation (operator that alters one or more values in an individual) during the generation of new individuals. The generated individuals are saved in the matrix  $\mathbf{B}_{Q\times 6M}$ . Every row of this matrix represents one individual. We seek the individual  $y_{\min} \in C$  satisfying the condition

$$F(y_{\min}) = \min\{F(y); y \in C\},\tag{7}$$

where  $C \subset E_{6M}$  is the searched set. Every element of C is formed by a set of 6M allowable parameters and this set defines just one constellation of the heaters above the mould. The identification of the individual  $y_{\min}$  defined by (7) is not realistic in practice. But we are able to determine an optimized solution  $y_{opt}$ . Now we describe particular steps of the genetic algorithm that is used.

Genetic algorithm

Input: the specimen  $y_1$  (initial individual),  $\varepsilon_1$  - the specified accuracy of the calculation.

Internal computation:

- 1. create an initial population of Q individuals,
- 2.a/ evaluate all the individuals of the population (calculate F(y) for every individual y),
- b/ sort values F(y) of all individuals y into ascending order and organize individuals y accordingly,
- c/ store the individuals y into the matrix B,
- 3. repeat until  $\min\{F(y); y \in \mathbf{B}\} < \varepsilon_1$ ,
- a/ choose randomly between the crossover operation and the mutation operation,
- b/ if the crossover operation is chosen then

randomly select (so-called roulette-wheel selection) a pair of individuals (parents), execute the crossover operation and create two new individuals else

randomly select (roulette-wheel selection) an individual y, execute the mutation operation, create two new individuals end if,

c/ calculate F(y) for the two new individuals (penalize an individual in the case of the collision of heaters or the collision of a heater and the mould surface), d/ sort as in step 2.b/, e/ take the first Q individuals y with the smallest values F(y) and store them in the matrix  $\mathbf{B}$  end repeat.

Output: the first row of matrix **B** contains the best found individual.

## 4. Use of parallel programming during the calculation

Some numerical solutions to practical examples of heat radiation intensity optimization, including graphical representation of the locations of the infrared heaters above the outer mold surface, are published in articles [4] and [5].

This section focuses exclusively on the possibility to use parallel programming to accelerate the optimization procedure. Optimization of locations of heaters using the genetic algorithm requires a number of numerical operations. The calculation of aberration F(y) or  $\tilde{F}(y)$  given by the relations (5) and (6) respectively is computationally the most demanding part of the genetic algorithm and is performed for every created new individual y (where value F(y) defines the fitness of the individual y). Before determining the value F(y) or  $\tilde{F}(y)$ , we have to perform the following computational steps for a given individual y (one of the possible locations of heaters): (i) for every heater  $Z_i$  ( $1 \le i \le M$ ) determine the heat radiation intensity over all elementary surfaces  $p_j$  ( $1 \le j \le N$ )(use the relation (3) for  $p_j$ , interpolate the value of the heat radiation intensity of heater  $Z_i$  on the  $p_j$  using the interpolation formula, (ii) calculate the total heat radiation intensity  $I_j$  on  $p_j$  using the relation (4) for

all  $p_j$ . The calculations of heat radiation intensities of the heaters  $Z_i$  and  $Z_k$  ( $i \neq k$ ) for each and every elementary surfaces  $p_j$  are completely independent. The calculation time of F(y) or  $\tilde{F}(y)$  and thus the overall time of the optimization procedure can be significantly reduced by using the tools of parallel programming. For this experiment we used a PC with 3GB RAM, CPU 2x AMD Athlon 64 X2 Dual Core 2.81 GHz. We performed experiments for one, two and four processors.

The tests are carried out for the aluminium mould of a passenger car dashboard (see Figure 1). The volume of the mould was  $0.6 \times 0.4 \times 0.12 \,\mathrm{m}^3$ , the number of elementary surfaces was N=2178. The infrared heaters used were all the same type (capacity  $1600 \,\mathrm{W}$ , length  $15 \,\mathrm{cm}$ , width  $4 \,\mathrm{cm}$ ), the manufacturer of artificial leathers recommended heat radiation intensity  $I_{rec}=47 \,\mathrm{kW/m^2}$ . The calculations were performed using a Matlab code (including parallel programming) written by the authors. First, we focused on the real time of the calculation of the aberration F(y) defined by relation (5). Real times of the calculation of F(y) for different numbers of processors used and different numbers of heaters are presented in Table 1. The times in Table 1 required to the calculation of F(y) were measured on the specified computer.

The total duration of the optimization procedure depends on the number of processors used, the number of heaters used and on the number of iterations of the genetic algorithm (two new individuals are generated in one iteration). The results are presented in Table 2. The maximum number of iterations in our tests was 100 000. We did not obtain better optimized solution after using a higher number of iterations. The times in Table 2 of total duration of optimization were measured as in Table 1 on the specified computer.

The duration of the optimization procedure can be significantly accelerated by using parallel programming to calculate F(y) as is shown in Table 2. The acceleration of the optimization procedure is effective especially with higher number of infrared heaters and large number of elementary surfaces of the mould surface.

Number	Number of used processors				
of applied	1	2	4		
heaters	Time of value $F(y)$ calculation [s]				
10	0.2510	0.1447	0.0915		
20	0.4928	0.3129	0.2229		
30	0.7853	0.4463	0.2769		
40	1.0470	0.5951	0.3692		
50	1.3088	0.7439	0.4615		

Table 1: Time required for the calculation of value F(y).

Number	Number	Number of used processors		
of applied	of GA	1	2	4
heaters	iterations	Time of optimization [h]		
10	20,000	1.3336	0.8366	0.5882
30	20,000	4.2792	2.6805	1.8812
50	20,000	7.2732	4.5499	3.1882
10	50,000	3.3340	2.0916	1.4704
30	50,000	10.6979	6.7016	4.7030
50	50,000	18.1831	11.3747	7.9706
10	100,000	6.6680	4.1832	2.9408
30	100,000	21.3958	13.4027	9.4061
50	100,000	36.3662	22.7495	15.9412

Table 2: Time required for the optimization procedure.

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