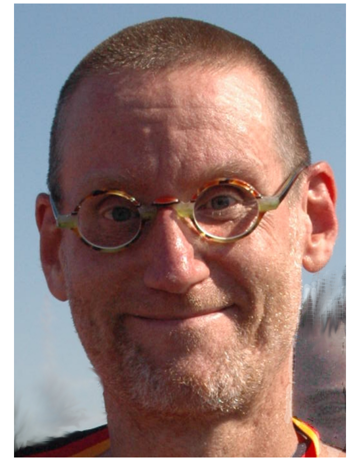
Jan Brandts



From Binary Cube Triangulations

to Acute Binary Simplices

MK-60

From Binary Cube Triangulations

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MK-40

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MK-21

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http://www.faceofthefuture.org.uk/

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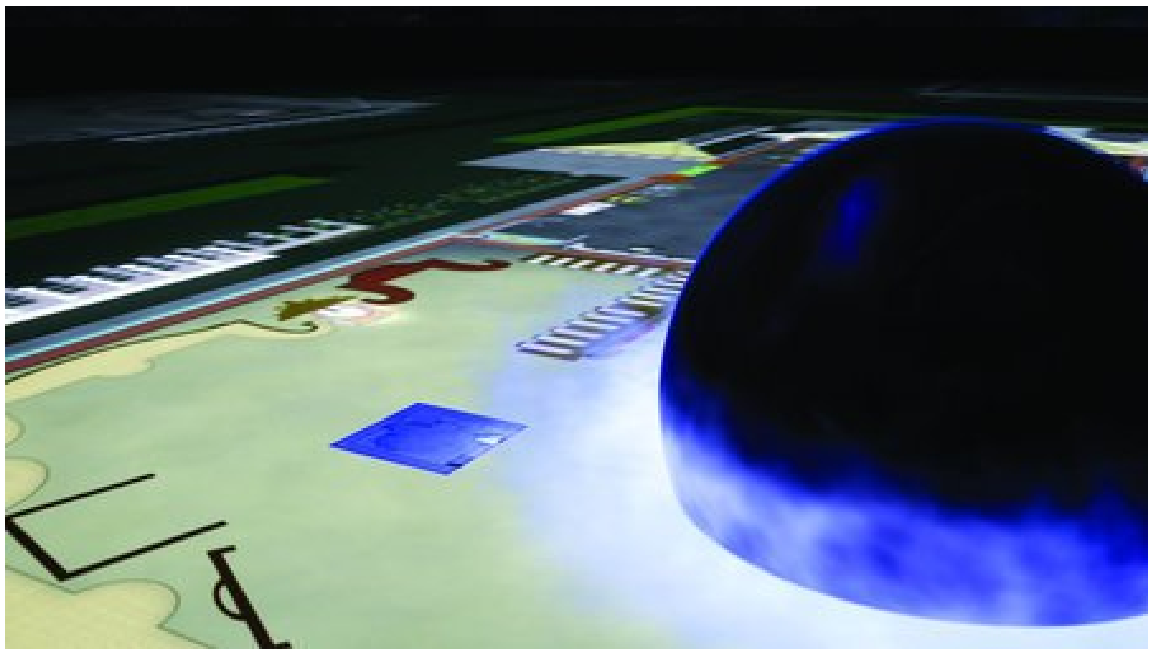
In Edwin Abbott's novel Flatland ...



... Spherius teaches Arthur Square about higher

In Edwin Abbott's novel Flatland ...

... Spherius teaches Arthur Square about higher


D I M E N S I O N S


Higher dimensional finite elements

Superconvergence:

- for quadratic tetrahedral elements
- for linear simplicial elements (unification, extension)

Discrete maximum principles: reaction-diffusion equations

- for linear simplicial elements

SIAM Review 2009

On nonobtuse simplicial partitions

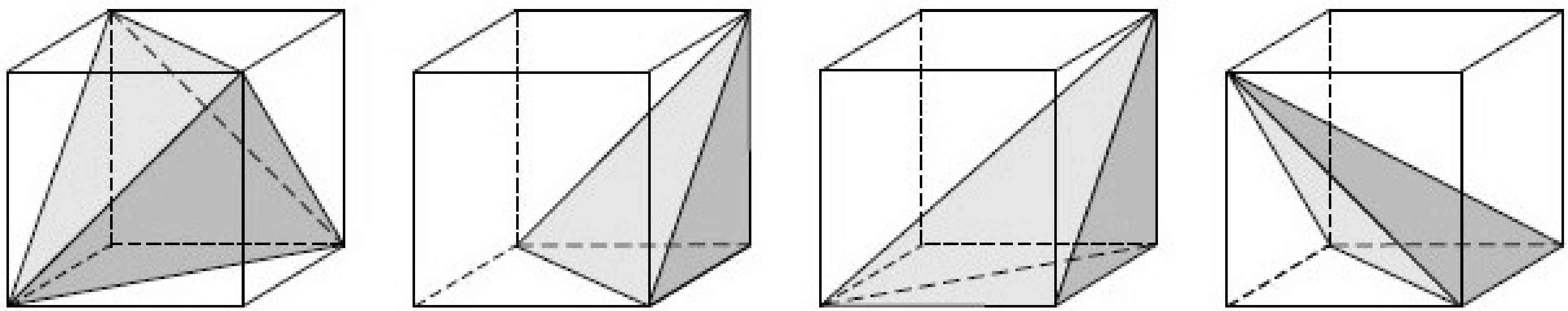
(with Sergey Korotov and Jakub Šolc)

Theme: Geometry Intertwining Numerical Analysis

- construction/refinement of nonobtuse and acute triangulation
- for FEM applications, but also *curiosity driven*

What is a nonobtuse or acute simplex?

• an angle between two $(n-1)$-facets of a simplex is *dihedral*



• a *nonobtuse* simplex has no *obtuse* $(> 90^o)$ dihedral angles

• an *acute* simplex has only *acute* $(< 90^o)$ dihedral angles

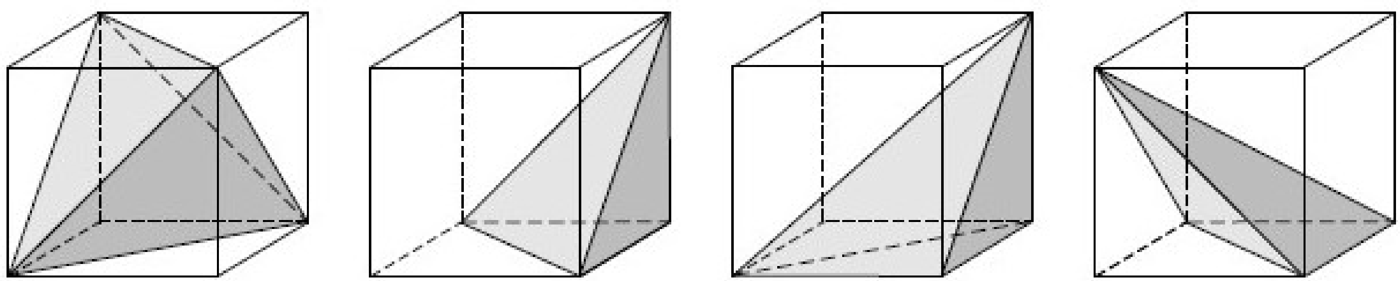
Research questions:

Typical quesions on the border between geometry and numerical analysis are, for instance:

- can the cube be acutely triangulated? (solved in 2009/2010)
- is there an acute triangulation of $\mathbb{R}^4$?
- can each simplex be decomposed into path-simplices?
- can each tetrahedron be triangulated with path-simplices?

0/1-**simplex, binary simplex**

A *binary* or 0/1-*simplex* shares its vertices with the unit n-cube



- simplest example of a 0/1-polytope (Ziegler et al)
- there are 2^{2^n} 0/1-polytopes in n dimensions
- some are the same modulo hyper-octahedral group action

Research questions about $0/1$-polytopes

- Minimal n-cube triangulation (solved for $n \leq 7$)
- Number of nondegenerate full-dimensional 0/1-polytopes
- Same, modulo cube symmetries
- Maximal number of facets of a 0/1-polytope

Many of these questions are very hard, unsolved, and (Ziegler)

"low dimensional intuition does not work"

Minimal n-cube triangulation simplified

n	1	2	3	4	5	6	7
#	1	2	5	16	67	308	1427

The total number of 0/1 simplices in the n cube is approximately

$$\binom{2^n}{n+1}$$

which for $n = 8$ equals approximately 10.000.000.000.000.000.

Instead, we tried to use only *nonobtuse* binary simplices, of which there are much, much less

Minimal n-cube triangulation simplified

Minimal nonobtuse binary cube triangulation: solved completely!

n	1	2	3	4	5	6	7	8
#	1	2	5	16	67	308	1427	
#	1	2	5	18	87	518	3621	28962

Cardinality of the minimal *nonobtuse* binary triangulation

$$N(n) = nN(n-1) - n + 2 \text{ with } \quad N(3) = 5$$

$$N(n) = 1 + n! \sum_{k=2}^{n} \frac{1}{k!}$$

Hadamard Maximum Determinant Problem

What is the maximum determinant of an $n \times n$ 0/1-matrix?

n	2	3	4	5	6	7	8	9	10	11	12	13
det	1	2	3	5	9	32	56	144	320	1458	3645	9477

For $n = 3$ mod 4 there is a formula: *Hadamard's Conjecture*

For 0/1-matrices representing *acute binary simplices*:

n	2	3	4	5	6	7	8	9	10	11	12	13
det	1	2	3	5	9	32	56	96	224	1458	3645	7290

Acute binary simplices

What is the structure of acute binary simplices? There are only few:

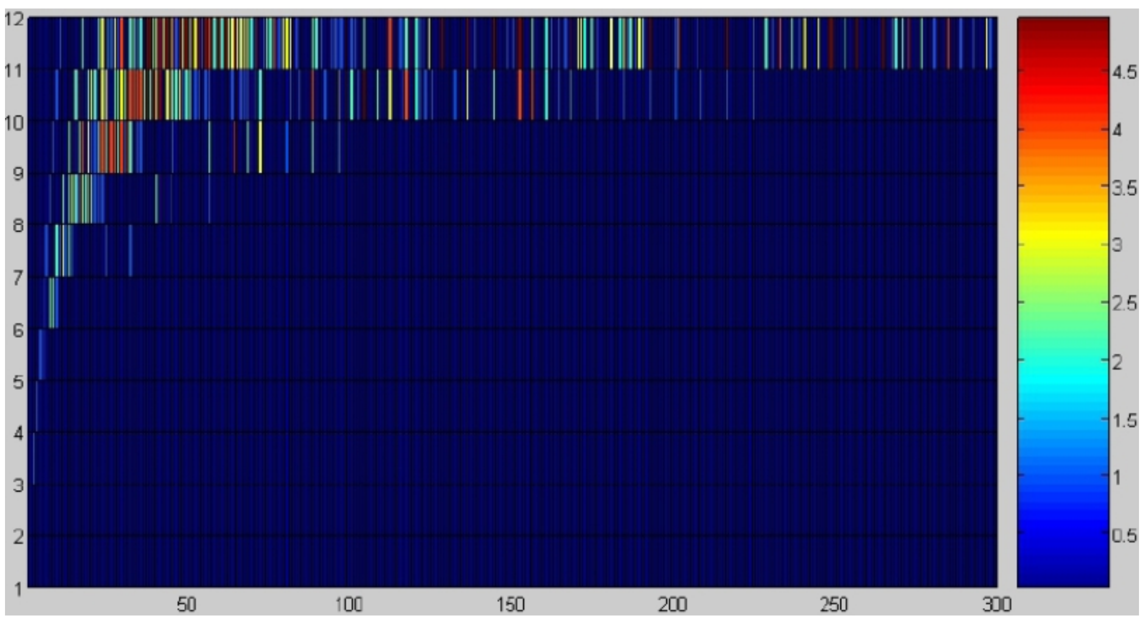
n	3	4	5	6	7	8	9	10	11
#	1	1	2	6	13	29	67	162	392

Amount of acute binary simplices modulo cube symmetries

Algebraically: $n \times n$ 0/1 matrix P represents an acute binary simplex if and only if the *dihedral angle information matrix* $(P^\top P)^{-1}$ is *diagonally strictly dominant, strictly Stieltjes, matrix*

Determinant spectrum acutely restricted

What is the range of the determinant function on such matrices?



Ongoing research

PhD project (2012-2016) for Apo Cihangir (Univ. Amsterdam)

- Nonobtuse triangulation of 0/1-polytopes
- Exhaustive enumeration of acute binary simplices
- Diagonally dominant Stieljes matrices
- Ultrametric matrices
- Lattice simplices and polytopes