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THE OPTIMIZATION OF HEAT RADIATION INTENSITY

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Abstract

This article focuses on the problem of calculating the intensity of heat radiation and its optimization across the surface of an aluminium or nickel mould. The inner mould surface is sprinkled with a special PVC powder and the outer mould surface is warmed by infrared heaters located above the mould. In this way artificial leathers are produced in the car industry (e.g., the artificial leather on a car dashboard). The article includes a description of how a mathematical model allows us to calculate the heat radiation intensity across the mould surface for every fixed location of the heaters. In calculating the intensity of the heat radiation, we use experimentally measured values of the heat radiation intensity by a sensor at the selected points in the vicinity of the heater. It is necessary to optimize the location of the heaters to provide approximately the same heat radiation intensity across the whole mould surface during the warming of the mould (to obtain a uniform material structure and colour tone of the artificial leather). The problem of optimization is more complicated (used moulds are often very rugged, during the process of optimization we avoid possible collisions of two heaters as well as of a heater and the mould surface). A genetic algorithm and the technique of hill climbing are used during the process of optimization. The calculations were performed by a Matlab code written by the authors. The article contains a practical example.

1. Introduction

This article describes a procedure for the calculation of radiation intensity across the whole mould surface for fixed locations of infrared heaters and the process of heat radiation intensity optimization on the mould surface. The problem of optimization is rather complicated, a genetic algorithm and the hill climbing technique are used to find suitable locations for the heaters over the mould and to optimize the heat radiation intensity across the whole outer mould surface.

2. A mathematical model of heat radiation

In this chapter a simplified mathematical model of heat radiation produced by infrared heaters and absorbed by the outer mould surface is described. The heaters and the heated mould are represented in 3-dimensional Euclidean space using the Cartesian coordinate system (O, x_1, x_2, x_3) with basis vectors $e_1 = (1, 0, 0)$, $e_2 = (0, 1, 0)$, $e_3 = (0, 0, 1)$.

Representation of a heater. A heater is represented by a line segment of length d. The location of a heater is defined by the following parameters: 1/ coordinates of the heater centre $S = [x_1^S, x_2^S, x_3^S]$, 2/ the unit vector $u = (x_1^u, x_2^u, x_3^u)$ of the heat radiation direction, where $x_3^u < 0$ (i.e., heater radiates "downward "), 3/ the vector of the heater axis $r = (x_1^r, x_2^r, x_3^r)$ (see Figure 1). The other way to determine r is by using the angle φ between the positive part of the x_1-axis and the vertical projection of r onto the x_1x_2-plane (the vectors u and r are orthogonal, $0 \le \varphi < \pi$). The location of every heater Z can be defined by the following 6 parameters

$$Z : (x_1^S, x_2^S, x_3^S, x_1^u, x_2^u, \varphi). \tag{1}$$

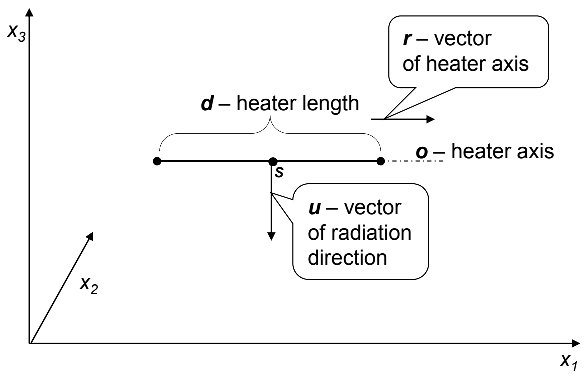

Figure 1: Representation of the heater in the model.

Representation of a mould. The mould surface P is described by the elementary surfaces p_j, where $1 \le j \le N$. It holds that $P = \cup p_j$, where $1 \le j \le N$ and $int\, p_i \cap int\, p_j = \emptyset$ for $i \ne j$, $1 \le i, j \le N$. Every elementary surface p_j is described by the following parameters: 1/ the center of gravity $T_j = [x_1^{T_j}, x_2^{T_j}, x_3^{T_j}]$, 2/ the unit outer normal vector $v_j = (x_1^{v_j}, x_2^{v_j}, x_3^{v_j})$ at the point T_j (we suppose v_j faces "upwards" and therefore is defined through the first two components $x_1^{v_j}$ and $x_2^{v_j}$), 3/ the size of its area s_j. Every elementary surface thus can be defined by the following 6 parameters:

$$p_j : (x_1^{T_j}, x_2^{T_j}, x_3^{T_j}, x_1^{v_j}, x_2^{v_j}, s_j). \tag{2}$$

3. The calculation of heat radiation intensity

We describe the process of calculating the heat radiation intensity on the mould surface for given fixed locations of the heaters. The heater manufacturer has not provided the distribution function of the heat radiation intensity in the heater surroundings. We realized the experimental measurement of the heat radiation intensity. The measured heater location was $Z:(0,0,0,0,0,0)$ in accordance with the relation (1), i.e., the center S of the heater was situated at the origin of the Cartesian coordinate system (O,x_1,x_2,x_3), the unit radiation vector had coordinates $u=(0,0,-1)$ and the vector of the heater axis had coordinates $r=(1,0,0)$. We suppose the heat radiation intensity across the elementary surface p_j is the same as at the centre of gravity T_j. The heat radiation intensity at T_j depends on the position of this point (determined by the first three parameters in the vector p_j given by the relation(2)) and on the direction of the outer normal vector v_j at the point T_j (determined by the fourth and fifth parameters in the vector p_j given by (2)). The heat radiation intensity in the surroundings of the heater was experimentally measured by a sensor placed at chosen points below the heater. We use a linear interpolation function of 5 variables to continuously interpolate the measured heat radiation intensity in the vicinity of the heater Z (for more detail see [4]).

For a heater in a general position, we briefly describe the transformation of the previous Cartesian coordinate system (O,e_1,e_2,e_3) into a positively oriented Cartesian system $(S,r,n,-u)$, where S is the centre of the heater, r is the heater axis vector, and u is the direction vector of the heat radiation. The vector n is determined by the vector product of the vectors $-u$ and r (see more detail in [1]) and is defined by the relation

$$n=(-u)\times r=\left(-\begin{vmatrix} x_2^u & x_3^u \\ x_2^r & x_3^r \end{vmatrix}, \begin{vmatrix} x_1^u & x_3^u \\ x_1^r & x_3^r \end{vmatrix}, -\begin{vmatrix} x_1^u & x_2^u \\ x_1^r & x_2^r \end{vmatrix}\right).$$

The vectors r, u and n are normalized to have the unit length. Then we can define an orthonormal transformation matrix

$$\mathbf{A}=\begin{pmatrix} x_1^r & x_1^n & -x_1^u \\ x_2^r & x_2^n & -x_2^u \\ x_3^r & x_3^n & -x_3^u \end{pmatrix}.$$

Let us recall that, for the elementary surface p_j, the respective triples T_j and v_j represent its centre of gravity and its outer normal vector in the Cartesian coordinate system (O,e_1,e_2,e_3). If S is the triple representing (in (O,e_1,e_2,e_3)) the center of the heater that determines the coordinate system $(S,r,n,-u)$, then T_j and v_j are transformed as follows:

$$\left(T_j'\right)^T=\mathbf{A}^T\left(T_j-S\right)^T \quad \text{and} \quad \left(v_j'\right)^T=\mathbf{A}^T v_j^T,$$

where T_j' and v_j' are the coordinates in $(S,r,n,-u)$. In this way, we transform the general case to the measured case.

Now we describe the procedure of numerical computation for the total heat radiation intensity on the mould surface. We denote L_j as the set of all heaters radiating on the jth elementary surface p_j ($1 \leq j \leq N$) for the fixed locations of heaters, and I_{jl} the heat radiation intensity of the lth heater on the p_j elementary surface. Then the total radiation intensity I_j on the elementary surface p_j is given by the following relation (see in detail in [2])

$$I_j = \sum_{l \in L_j} I_{jl} \; . \tag{3}$$

The producer of artificial leathers recommends the constant value of heat radiation intensity across the whole outer mould surface. Let us denote this constant value as I_{rec}. We can define F, the aberration of the heat radiation intensity, by the relation

$$F = \frac{\sum_{j=1}^{N} |I_j - I_{rec}| s_j}{\sum_{j=1}^{N} s_j} \tag{4}$$

and the aberration $\tilde{F}$ by the relation

$$\tilde{F} = \sqrt{\sum_{j=1}^{N} \left(I_j - I_{rec}\right)^2 s_j} \; . \tag{5}$$

4. The optimization of the location of the heaters

We use a genetic algorithm for global optimization and subsequently the hill climbing method for local optimization of the locations of heaters. These methods are described in more details in [3] and in [5]. The location of every heater is defined in accordance with the relation (1) by 6 parameters. Therefore $6M$ parameters are necessary to define the locations of all M heaters. One chromosome represents one individual (one possible location of the heaters). The population includes Q individuals. The generated individuals are saved in the matrix $\mathbf{B}_{Q\times 6M}$. Every row of this matrix represents one individual. We seek the individual $y_{min} \in C$ satisfying the condition

$$F(y_{min}) = min\{F(y); y \in C\}, \tag{6}$$

where $C \subset E_{6M}$ is the searched set. Every element of C is formed by a set of $6M$ allowable parameters and this set defines just one constellation of the heaters above the mould. The function F is defined by (4) or by (5). The identification of the individual y_{min} defined by (6) is not realistic in practice. But we are able to determine an optimized solution y_{opt}. Now we describe particular steps of the genetic algorithm that is used.

Genetic algorithm

Input: the specimen y_1 (initial individual), ε_1 - the specified accuracy of the calculation.

Internal computation:
1. create an initial population of Q individuals,
2.a/ evaluate all the individuals of the population (calculate $F(y)$ for every individual y), b/ sort $F(y)$ in the ascending order and organize y accordingly, c/ store the individulas y into the matrix $\mathbf{B}$,
3. *repeat until* $min\{F(y); y \in \mathbf{B}\} < \varepsilon_1$
a/ chose randomly between the crossover operation and the mutation operation,
b/ *if* the crossover operation is chosen *then*
randomly select a pair of individuals (parents),
execute the crossover operation and create two new individuals
else
randomly select an individual y, execute the mutation operation,
create two new individuals
end if,
c/ calculate $F(y)$ for the two new individuals (penalize an individual in the case of the collision of heaters or the collision of a heater and the mould surface), d/ sort as in 2.b/, e/ take the first Q individuals with the best evaluation $F(y)$ and store them in the matrix $\mathbf{B}$
end repeat.
Output: the first row of matrix $\mathbf{B}$ contains the best found individual.

To further optimize y_{opt} delivered by the genetic algorithm, we apply the hill climbing method.

Hill climbing algorithm
Input: y_{opt} - the solution provided by the genetic algorithm, real suitable increments h_i, where $1 \leq i \leq 6M$, ε_2 - the specified accuracy of the calculation.
Internal computation:
repeat until $max\{|h_i|; 1 \leq i \leq 6M\} < \varepsilon_2$
for i:= 1 *to* $6M$ *do*
a/ *while* $F(y_{opt}) > F(y_{opt} + h_i)$ *do* $y_{opt} := y_{opt} + h_i$
end while,
b/ $h_i := -h_i/2$
end for
end repeat.
Output: the best found individual.

The individual y_{opt} is the final optimized solution and includes information about the location of every heater in the form (1).

5. A practical example

Now we describe a practical example of the heating of an aluminium mould. The volume of the mould is $0.8 \times 0.4 \times 0.15[m^3]$, the number of elementary surfaces is $N = 2,064$; the recommended heat radiation intensity is $I_{rec} = 47[\mathrm{kW/m^2}]$. We use 16 infrared heaters of the same type (producer Philips, capacity $1,600$[W], length 15[cm], width 4[cm]). In the first step of our procedure we construct a specimen y_1 (this individual corresponds to the default locations of heaters). The centers of the heaters lie in the plane parallel to the x_1x_2-plane and at a distance of 10[cm] from the center of gravity T_j of the elementary surface p_j with the highest value $x_3^{T_j}$ $(1 \leq j \leq N)$. All the heaters have $r = (1,0,0)$ and $u = (0,0,-1)$ (that is, all the heaters radiate downwards and they are parallel to the axis x_1). The population contains 30 individuals ($Q = 30$).
For the initial specimen y_1 and F given by (4), we get $F(y_1) = 20.74$. We obtain the optimized individual y_{opt} with value $F(y_{opt}) = 3.39$ after $100,000$ iterations of the genetic algorithm and $5,000$ iterations of the hill climbing method (two individuls are generated during every iteration of the genetic algorithm and one individual is generated during every iteration of the hill climbing method). The value $F(y_{opt})$ depends on the number of iterations of the genetic algorithm and hill climbing method (see Figure 2).

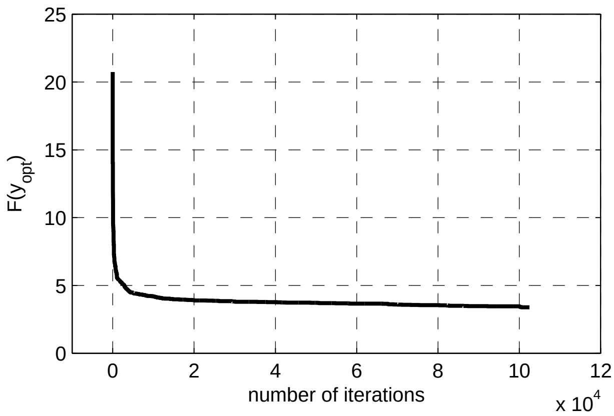

Figure 2: Dependence of $F(y_{opt})$ on the number of iterations.

The graphical representation of heat radiation on the mould surface (levels of radiation intensity in $[\mathrm{kW/m^2}]$ correspond to shades of grey colouring) and the locations of the heaters corresponding to the individual y_{opt} are displayed in Figure 3.

Let us replace F by $\tilde{F}$, see (5), and let us execute the same number of iterations. We get the following results: $\tilde{F}(y_1) = 25.13$; $\tilde{F}(y_{opt}) = 3.34$.
On the basis of our numerical tests, we have obtained results sufficiently accurate for the needs of production.

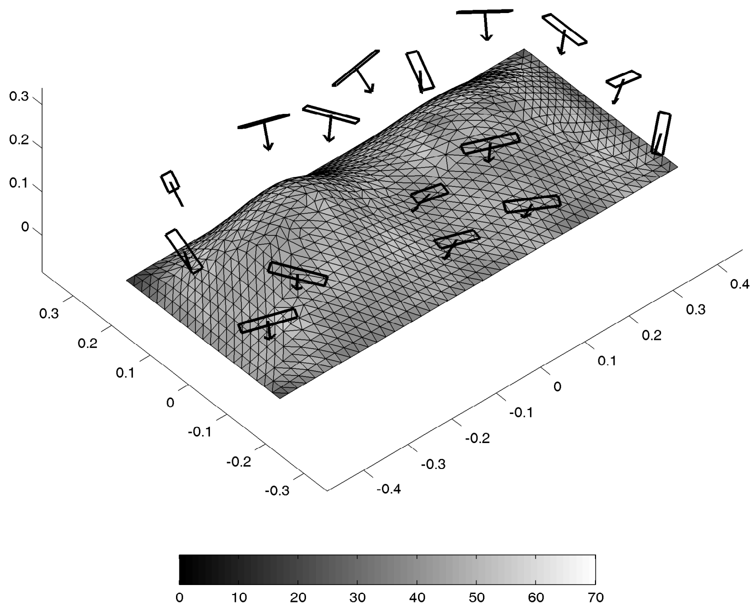

Figure 3: Heat radiation intensity([kW/m^2]) on the mould surface and the location of the heaters corresponding to the individual y_{opt}.

Acknowledgements

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