The Process of an Optimized Heat Radiation Intensity Calculation on a Mould Surface

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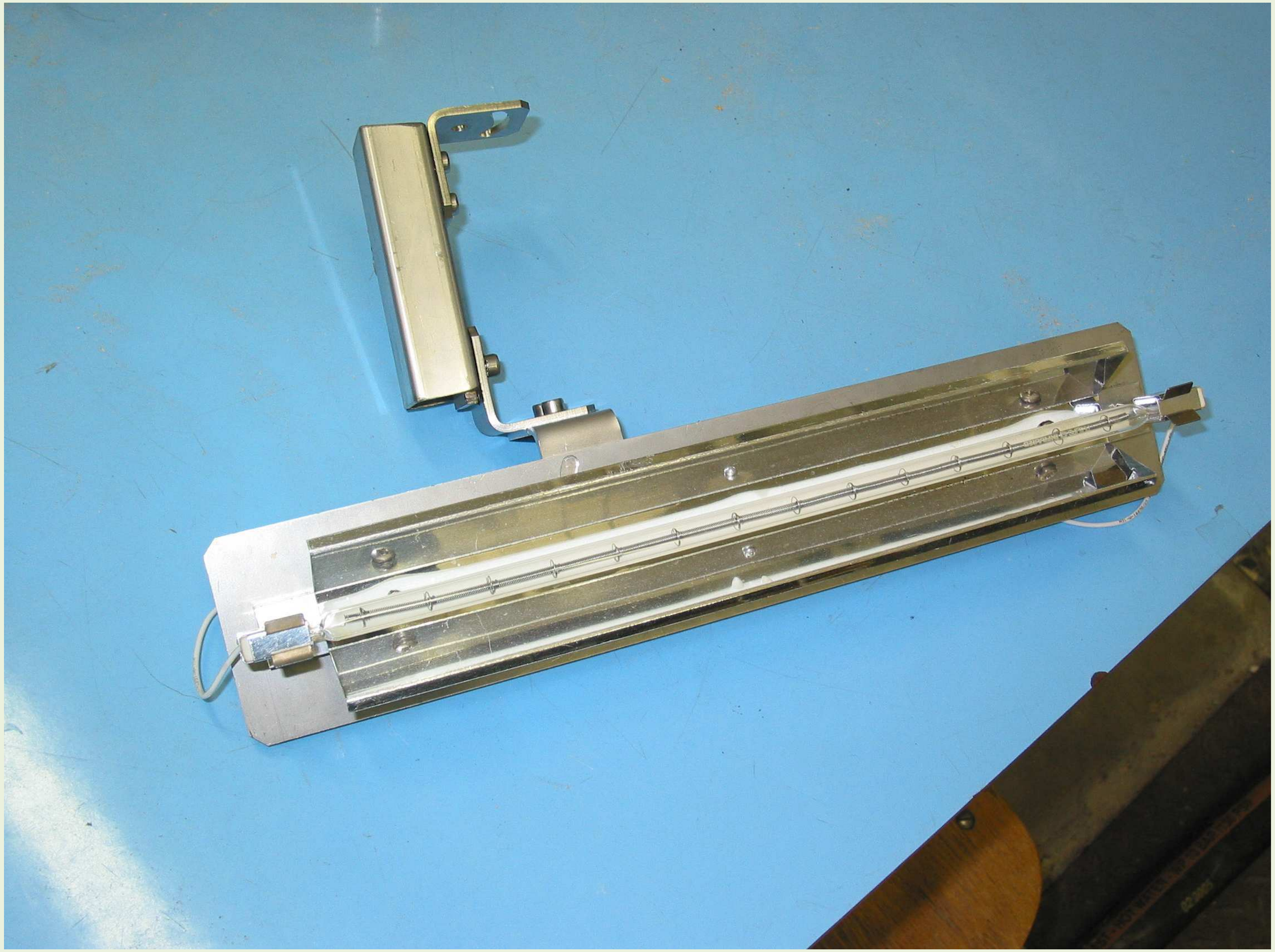
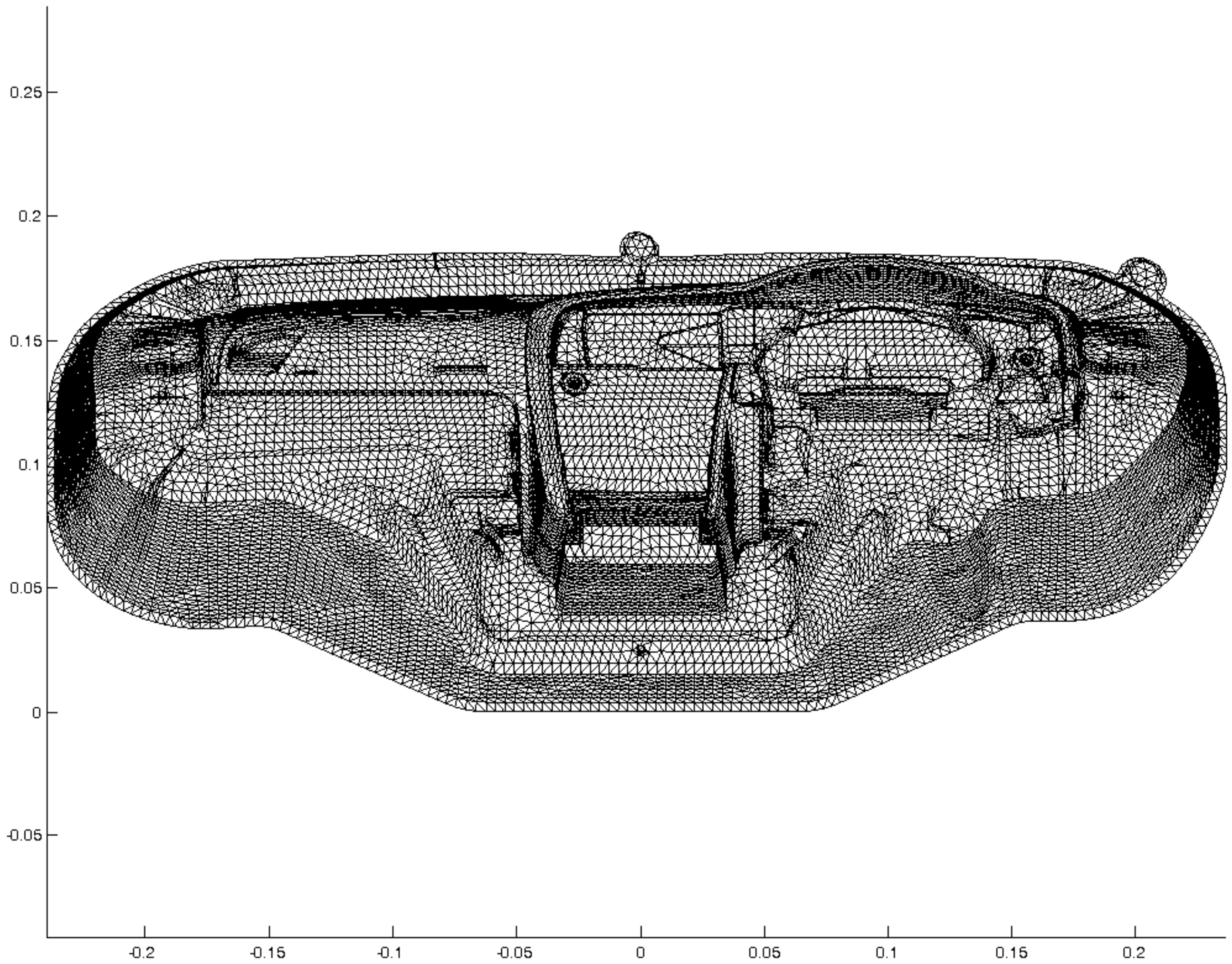
1. Introduction to the problem

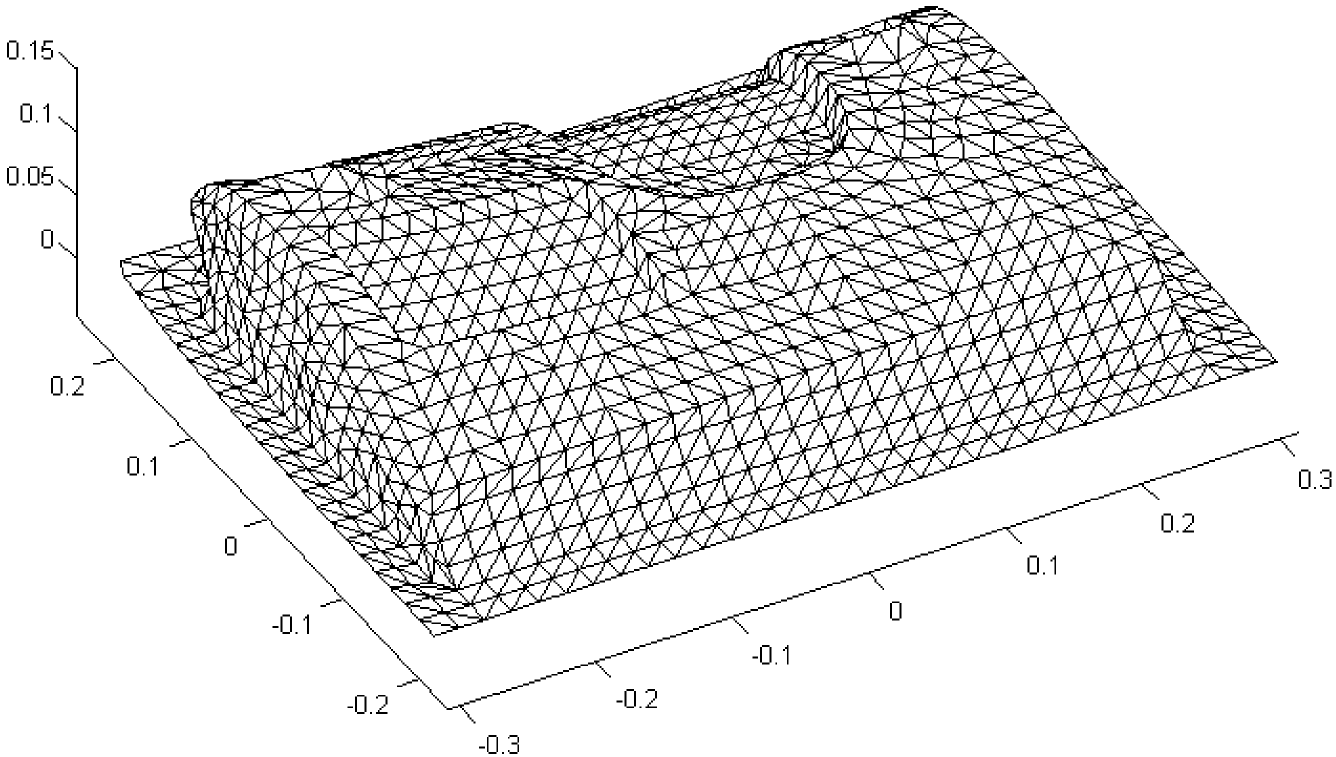
- This article is focused on the process of the heat radiation intensity optimization on an aluminium mould surface intended for the production of artificial leather in the car industry (e.g. the artificial leather on a car dashboard).
- A mould weigh is approximately 300kg.
- The mould is preheated and then the surface is sprinkled with a special PVC powder.
- Outside of the mould is heated by infra heaters located above the mould, up to a temperature of 250°C.
- It is necessary to carry out the configuration optimization of the infra heater locations above the mould in such way that the heat radiation intensity on the mould surface is approximately the same (within the given tolerance) - the same material structure and color of the artificial leather produced will thereby be obtained.
- Producer delivers artificial leathers to car companies Škoda (Mladá Boleslav, Czech Republic, member of Volkswagen Group) and Audi (especially for car Audi B8).



Philips infra heater with a 1000W capacity



1st example of a mould



2nd example of an aluminium mould - 0,6x0,4x0,12[m^3]

- Infra heaters have a tubular form and their length is usually between 15 and 30cm.
- The heater is equipped with a mirror located above radiating tube, which reflects back heat radiation in the adjusted direction.
- We use usually from 50 to 150 heaters to warming of mould surface in practice.
- A producer uses moulds of different sizes and moulds are often very rugged.
- The setting of the heater locations in production was adjusted upon the basis of the experience of technicians.
- The heater location and fixation of the heaters is laboured and time-consuming.
- We can’t use features of point heat radiation to solution of our problem.
- We also don’t know heater distribution function from the heater manufacturer.

We keep track of the following 3 types of possible heater collisions:

- one heater doesn’t radiate on a second heater more than the given limit,
- one heater has sufficient distance from second heater,
- a heater has sufficient distance from the mould surface and is over the surface.

The technical problem of optimization is rather complicated, we used a genetic algorithm and „hill-climbing“ method“ as the methods of reaching a solution.

We will use a genetic algorithm for global optimization (this method is less liable to get stuck in the local minimum) and upon finding a solution we will apply the „hill-climbing“ method to locally optimize the heater’s location.

2. Model of heat radiation on the mould circumference

- We will assume the representation of the heaters and mould

in a 3-dimensional Euclidean space E_3 with a coordonate systém.

<u>The mould surface</u>

- The mould surface is given by elementary surfaces p_j, where

$1 \leq j \leq N$.

- It holds $\bigcup_{1 \leq j \leq N} p_j = P$, where P denotes the total surface of the

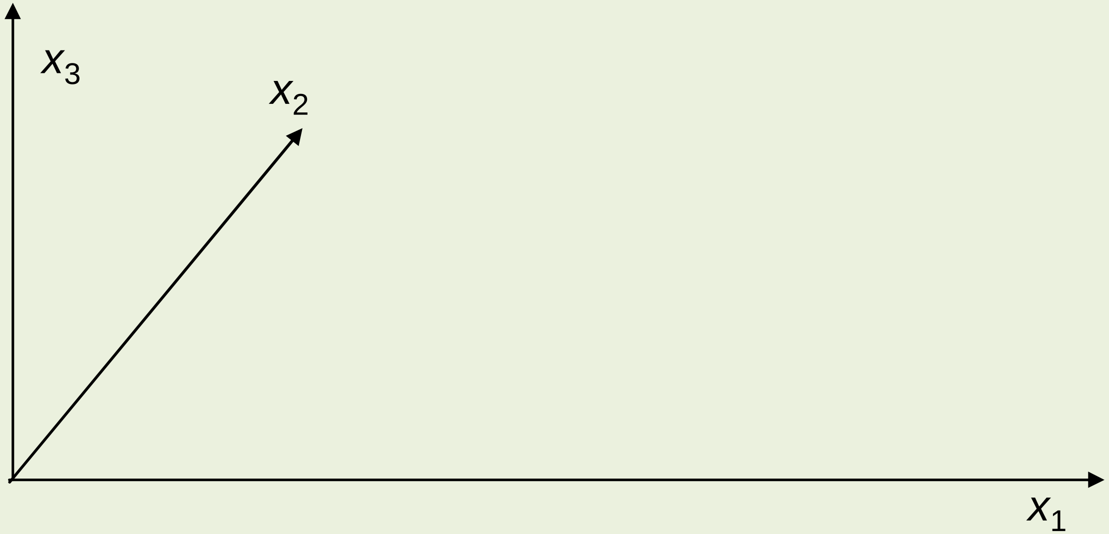
mould and $\operatorname{int} p_i \cap \operatorname{int} p_j = \emptyset$ for $i \neq j$.

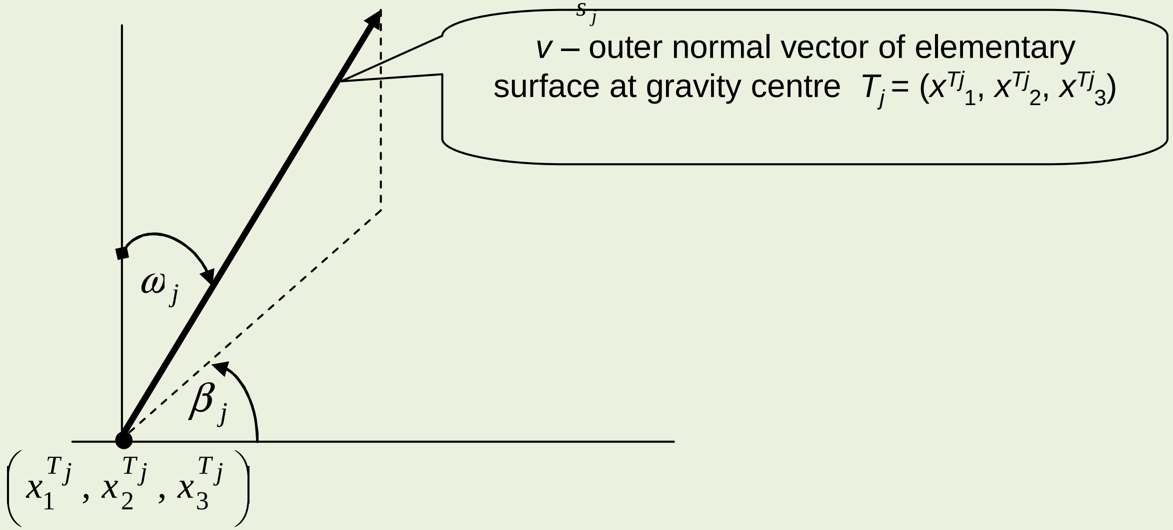
- Every elementary surface is represented by following parameters:

- its centre of gravity $T_j = [x_1^{T_j}, x_2^{T_j}, x_3^{T_j}]$,

- the outer normal vector $v_j = \left(x_1^{v_j}, x_2^{v_j}, x_3^{v_j} \right)$ at point T_j ,

- the area of elementary surface s_j [m^2].



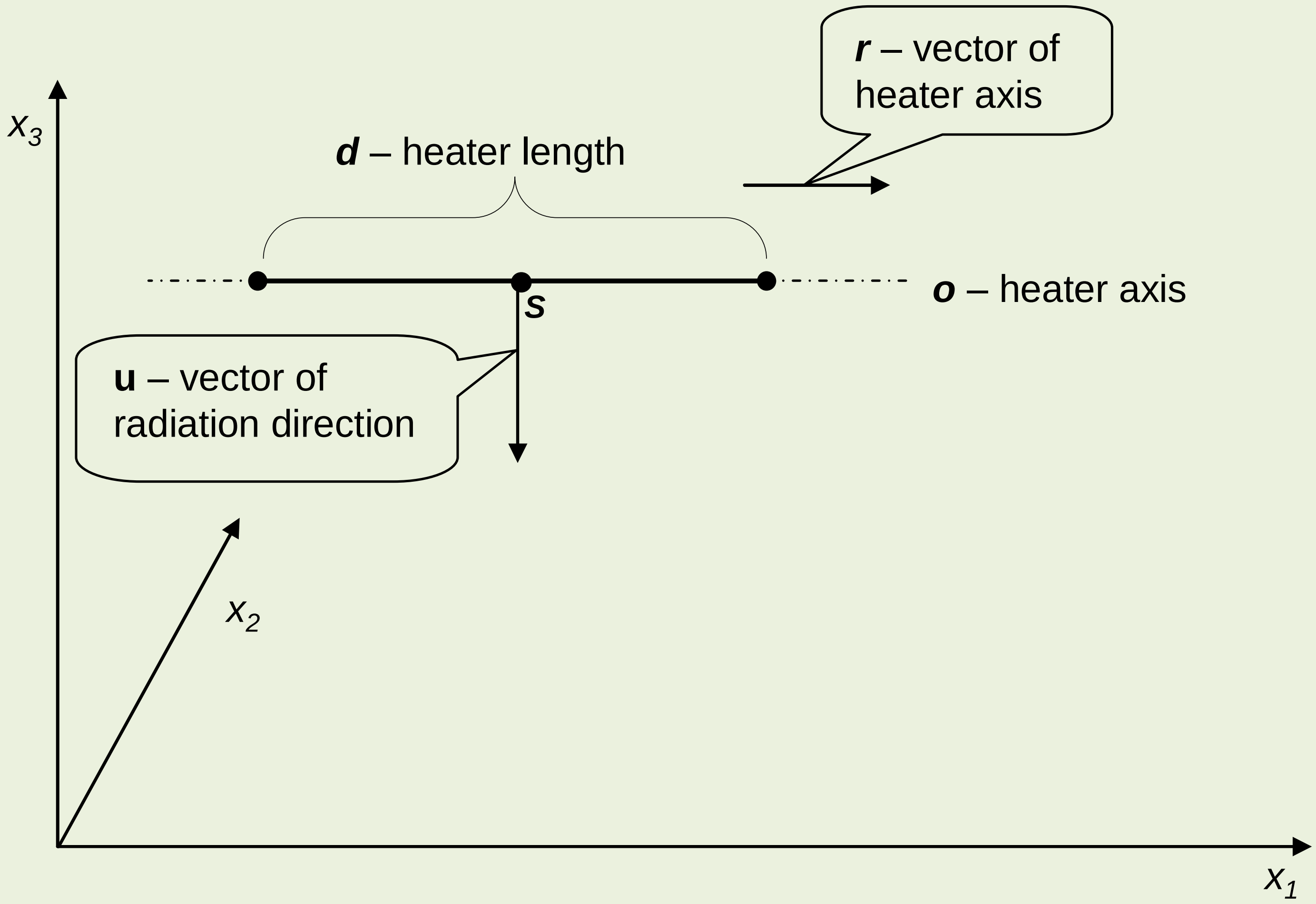


Parameters determining the elementary surface p_j.

- We suppose that the outer normal vector v_j at the point T_j has a unit length.
- It is then possible to define the vector v_j through angles β_j and ω_j , where positively oriented angle $0 \le \beta_j < 2\pi$ determines the size angle of the positive part of axis x_1 and the vertical projection of the outer normal vector v_j to a plane given by axes x_1 and x_2 (ground plane). The angle ω_j is determined by the outer normal vector and positive part of axis x_3 , where $0 \le \omega_j \le \pi / 2$.
- Every elementary surface is defined then by 6 parameters:

$$p_j : \left(x_1^{T_j}, x_2^{T_j}, x_3^{T_j}, \beta_j, \omega_j, s_j \right), \; 1 \le j \le N \quad . \qquad (1)$$

A heater



Schematic representation of the heater.

- We assume:
  - all heaters used have the same capacity and are of the same type,
  - every heater is represented by abscissa *d*[m] in length.

- The location of a heater is described by the following parameters:
  - coordinates of the heater centre $S = [\ x_1^S, x_2^S,\ x_3^S\ ]$,
  - radiation direction vector $u = \left(x_1^u, x_2^u, x_3^u\right)$, we assume a unit vector length *u* and we assume component x_3^u is negative, i.e. the heater radiates "down", then the coordinate x_3^u of vector *u* is explicitly allocated,

- The vector of the heater axis $r = \left(x_1^r, x_2^r, x_3^r\right)$: we assume that the length of the vertical projection of vector *r* to a plane given by the axes x_1 and x_2 (ground plane) is 1. This projection and positive part of axis x_1 define angle φ, where $0 \leq \varphi < \pi$, then $x_1^r = \cos\varphi$, $x_2^r = \sin\varphi$, component x_3^r is explicitly defined (vector *u* and *r* are orthogonal).

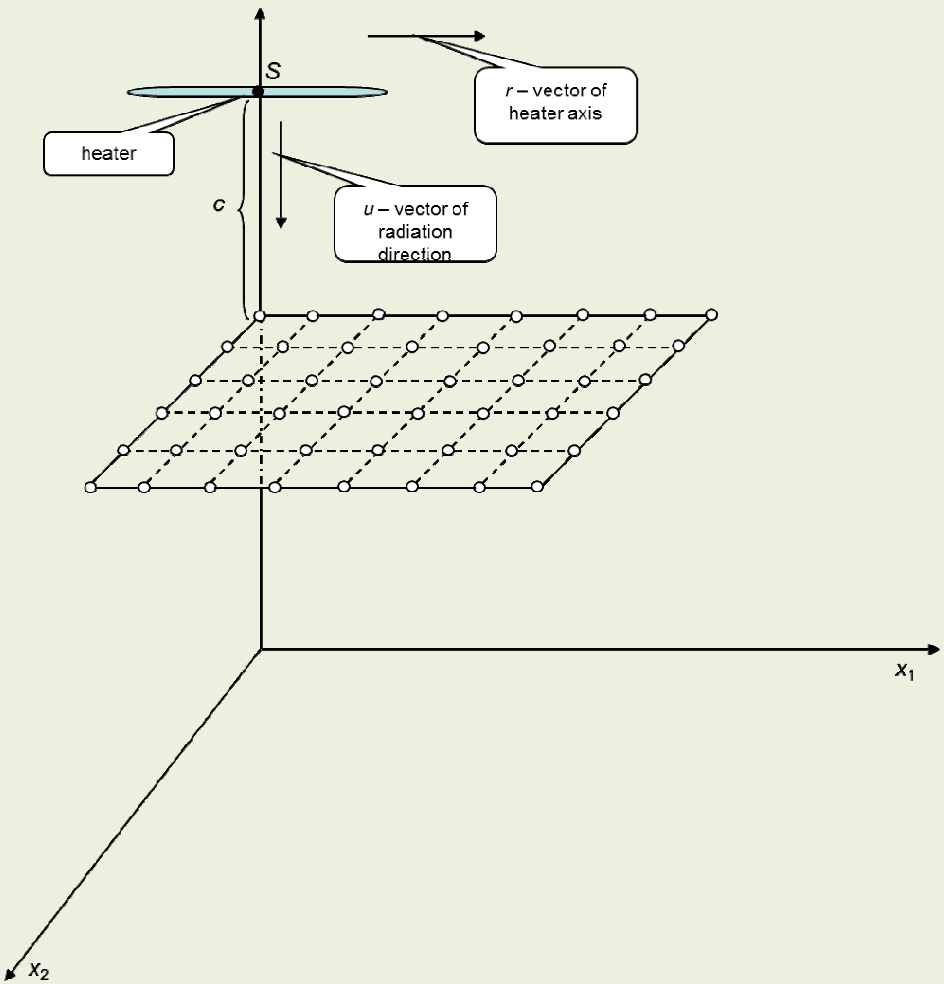
- The location of every heater Z is described by the following 6 parameters:

$$\text{Z:} \quad \left(x_1^S, x_2^S, x_3^S, x_1^u, x_2^u, \varphi\right) \quad . \qquad (2)$$

- The location of *M* heaters is described by 6*M* parameters.

3. Radiation intensity determination in heater surroundings

- We will need to know the radiation intensity in heater surroundings to calculate the radiation intensity on the mould surface.
- An experimental measurement was taken of the radiation intensity of a given type of heater for selected points in the vicinity of the heater with the aid of a checking member.



An experimental measurement of the radiation intensity by a sensor.

We will use linear interpolation function of 5 variables to determine the heat radiation intensity in the vicinity of heater *Z*.

General Case of a Heater Location

- We transform Cartesian coordinate system (O,e_1, e_2, e_3) into a positively oriented system (S, r, n, $-u$), where:
  - point S is the centre of heater *Z,*
  - *r* is a heater axis,
  - *u* is the vector of radiation direction of heater *Z,*
  - *n* vector is defined by the vector product of the vectors $-u$ and r

$$n = (-u) \times r = \left(-\begin{vmatrix} x_2^u & x_3^u \\ x_2^r & x_3^r \end{vmatrix}, \begin{vmatrix} x_1^u & x_3^u \\ x_1^r & x_3^r \end{vmatrix}, -\begin{vmatrix} x_1^u & x_2^u \\ x_1^r & x_2^r \end{vmatrix} \right).$$

- We assume the unit length of the vectors r, n, $-u$ and we define orthonormal matrix **A**

$$\mathbf{A} = \begin{pmatrix} x_1^r & x_1^n & -x_1^u \\ x_2^r & x_2^n & -x_2^u \\ x_3^r & x_3^n & -x_3^u \end{pmatrix}$$

- Let us denote $T_j' = [x_1^{T_j'}, x_2^{T_j'}, x_3^{T_j'}]$ the transformation of the centre gravity $T_j = [x_1^{T_j}, x_2^{T_j}, x_3^{T_j}]$ of an elementary surface p_j in Cartesian coordinate system $(S, r, n, -u)$.

- Let the vectors $\mathbf{T}_j$, $\mathbf{T}_j'$, $\mathbf{S}$ represent coordinates of the points T_j , T_j' and S . Then the transformation of the point T_j is given by relation

$$\left(\mathbf{T}_j'\right)^T = \mathbf{A}^T\left(\mathbf{T}_j - \mathbf{S}\right)^T .$$

- An analogy of the transformation of the outer normal vector v_j in the centre gravity T_j of elementary surface p_j is given by relation

$$\left(\mathbf{V}_j'\right)^T = \mathbf{A}^T \mathbf{V}_j^T .$$

- We convert a general case of heater location on the basis of these two relations to the case of heater location with realized experimental measurement of the surrounding heat radiation intensity.

4. Calculation of radiation intensity on the surface mould

- We denote L_j a set for all the heaters radiating on the *j*-th elementary surface for the defined location of the heaters, where $1 \leq j \leq N$. We denote I_{jl} [W/m²] radiation intensity of the *l*-th heater on the *j*-th elementary surface.
- Then the total radiation intensity I_j on the *j*-th elementary surface is defined by the relation

$$I_j = \sum_{l \in L_j} I_{jl} \quad . \qquad (3)$$

- I_{rec} is the recommended radiation intensity on the mould surface by the producer.
- Difference F_j of the radiation intensity I_j on the *j*-th elementary surface ($1 \leq j \leq N$) from the recommended intensity I_{rec} is defined upon the basis of the relation

$$F_j = \left| I_j - I_{rec} \right| \qquad (4)$$

and the average aberration of radiation intensity F is given the relation

$$F = \frac{\sum_{j=1}^{N} F_j s_j}{\sum_{j=1}^{N} s_j} \quad . \qquad (5)$$

We often use the aberration $\tilde{F}$ determined by relation

$$\tilde{F} = \left(\sum_{j=1}^{N} (I_j - I_{rec})^2 s_j \right)^{1/2} . \qquad (5a)$$

5. Optimization of heater locations by the use of a genetic algorithm

- We suppose that it is used *M* heaters for the heat radiation of a mould surface.
- The location of every heater is defined by 6 real parameters according to relation (2). Then 6*M* parameters are necessary to the definition of the locations of all the heaters.
- A population will contain *Q* individuals.
- Continuous generated individuals will be saved in the matrix $\boldsymbol{A}_{Q \times 6M}$. Every row of matrix ***A*** represents one individual.
- Our aim is to find such individual y , that radiation intensity on the mould surface approaches the value I_{rec} recommended by the producer, i.e. we seek individual $y_{\min} \in B$ satisfactory condition

$$F(y_{\min}) = \min_{y \in B} F(y) \quad , \qquad (6)$$

where $B \subset E_{6M}$ is searched space and function *F* is defined by relation (5).

Schematic description of a genetic algorithm:

1/ the creation of a specimen and a initial population of individuals,

2/ the evaluation of the all individuals, the execution of all individuals sorted according to their fitness *F,*

3/ **while** a condition of termination isn't fulfilled **do**

if operation crossover is randomly chosen **then**

randomly select a pair of parents,

execute of crossover

else

randomly select an individual,

execute of mutation

end if,

integration and evaluation (fitness) of new calculated individuals ,

sorting of all individuals in accordance with evaluations ,

storage only first *Q* individuals with the best evaluation for subsequent calculation

end while,

4/ output of the best individual found – 1. row of matrix ***A*** .

- The setting of the starting individual (specimen) is chosen in such a way that all the centres of heaters create nodes in a regular rectangular network and lie over the mould, in parallel plane with the plane defined by axes x_1 and x_2. We will consequently generate next *Q* -1 individuals by random modifications of genes values.
- The following functions and operations are used in the schematic description of a genetic algorithm:

  - *Evaluation function* (*fitness*) *F* for individuals is defined by relation (5).

  - *Testing procedure against collisions*: one heater doesn't radiate on a second heater more than the given limit, one heater has sufficient distance from the second heater, a heater has sufficient distance from mould surface and is over the surface. If a collision exists, individual *y* is penalized and consequently expelled from population.

  - *Individual selection* to crossover or to mutation is implemented on the principle *fitness-proportionate selection*.

We will specify

$$G = \sum_{j=1}^{Q} \frac{1}{F(y_j)} \ , \tag{7}$$

where *F* is the evaluation function defined by relation (5). We suppose that the radiation intensity of heaters on the surface of the mould isn't wholly uniform when the location of heaters are represented by individual y_j, i.e. $F(y_j) \neq 0$ for $j = 1, 2, \dots, Q$. The probability of selection of individual y_i is defined by relation

$$p(y_j) = \frac{1}{F(y_j) \cdot G} \ . \tag{8}$$

We generate a random number $r \in < 0, 1 >$ in every selection of an individual from the population. We will select an individual y_i if and only if the following holds true

$$\sum_{j=1}^{i-1} p(y_j) < r \leq \sum_{j=1}^{i} p(y_j) \quad \text{for} \quad i = 1, 2, \dots, Q, \tag{9}$$

where we put $\sum_{j=1}^{i-1} p(y_j) = 0$ for $i = 1$.

During *operation crossover* we do only *one point crossover* and modify variants of crossover.

We also do operation *mutation*.

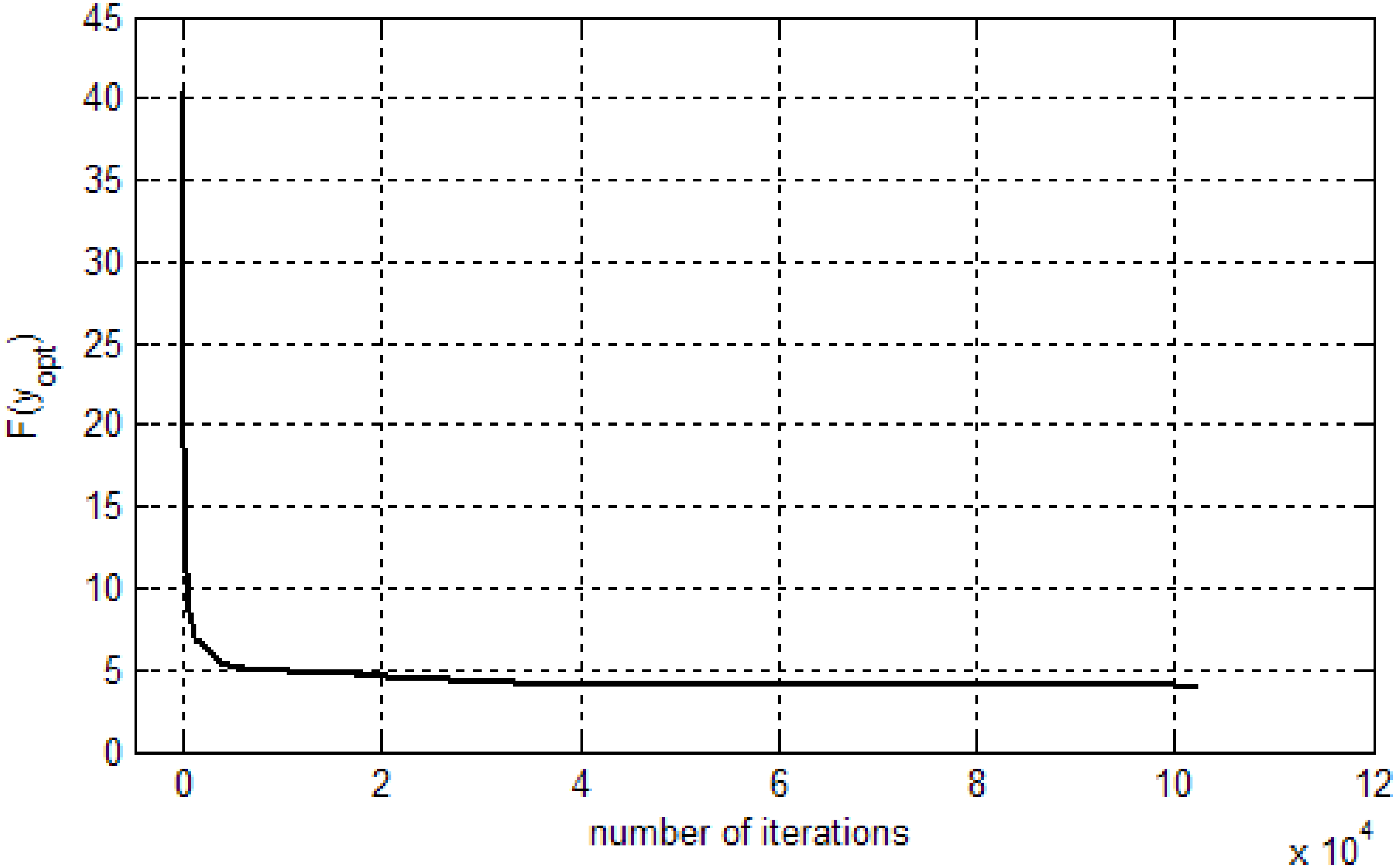
Algorithm of „hill-climbing“ method is described in article.

6. Examples of radiation intensity optimization

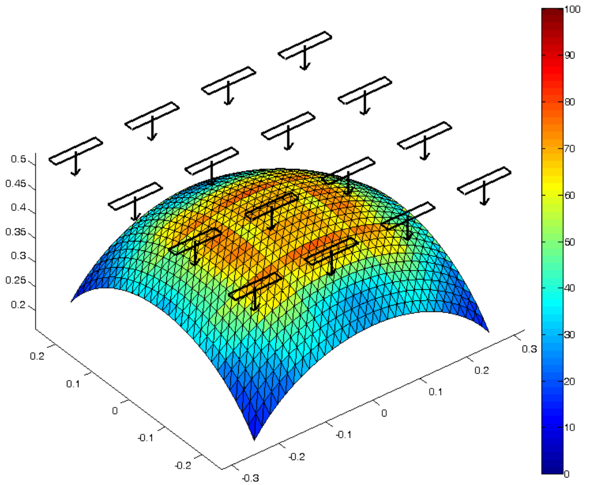
- We will describe the results of the radiation intensity optimization calculation on the mould surfaces for a given moulds.
 A software application was programmed in the Matlab language.
The parameters of all the heaters were the following: *producer Philips, capacity 1600W, length 15cm, width 4cm*.

- **Example 1**
- spherical surface, the radius of the sphere is 0,4[m],
- described by 1800 triangular elementary surfaces,
- I_{rec}=68[kW/m^2],
- we used 16 heaters
- 100 000 iterations of GA, 5000 iterations of „hill-climbing”

$F(y_1)=40{,}46$; $F(y_{opt})=3{,}86$



45
40
35
30
25
20
15
10
5
0
F(y_opt)
0
2
4
6
8
10
12
number of iterations
x 10^4



100
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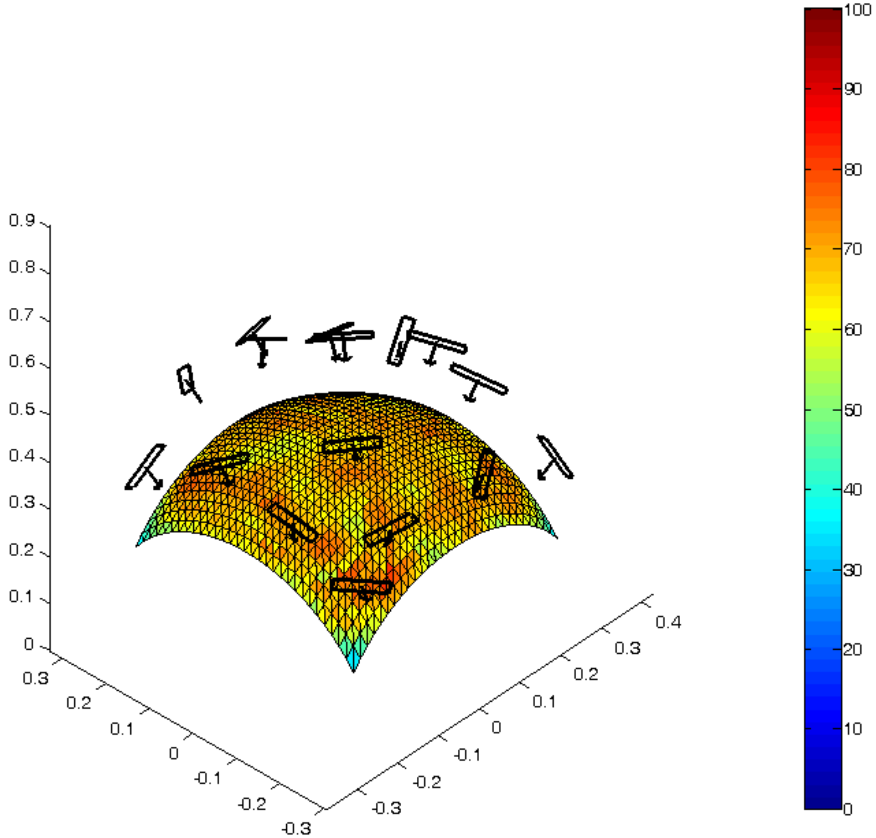

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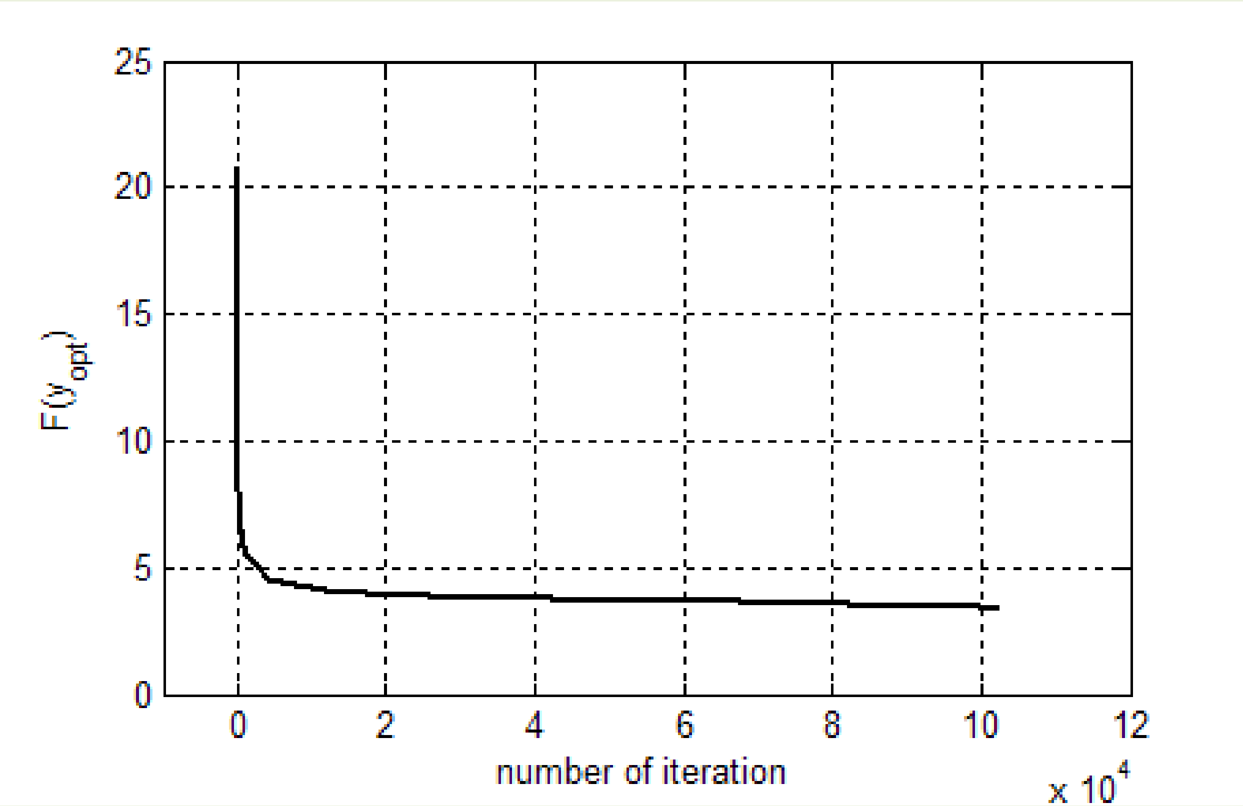
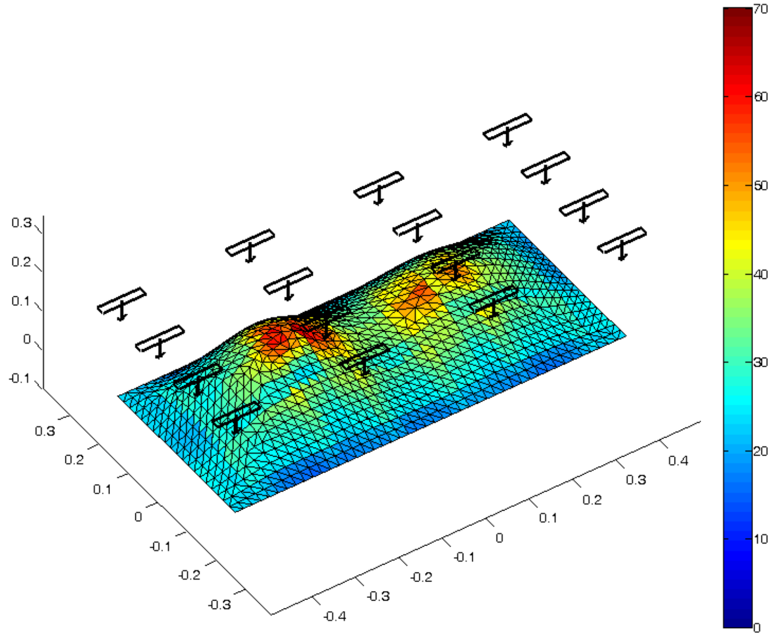
-0.3
-0.2
-0.1
0
0.1
0.2
0.3
0.4

- **Example 2**
  - the size of the mould is 0,8x0,4x0,12[m^2]
  - 2064 elementary triangular surfaces
  - I_{rec}=47[kW/m^2],
  - we used 16 heaters
  - 100 000 iterations of GA, 5000 iterations of „hill-climbing”

  $F(y_1)$=20,74; $F(y_{opt})$=3,39





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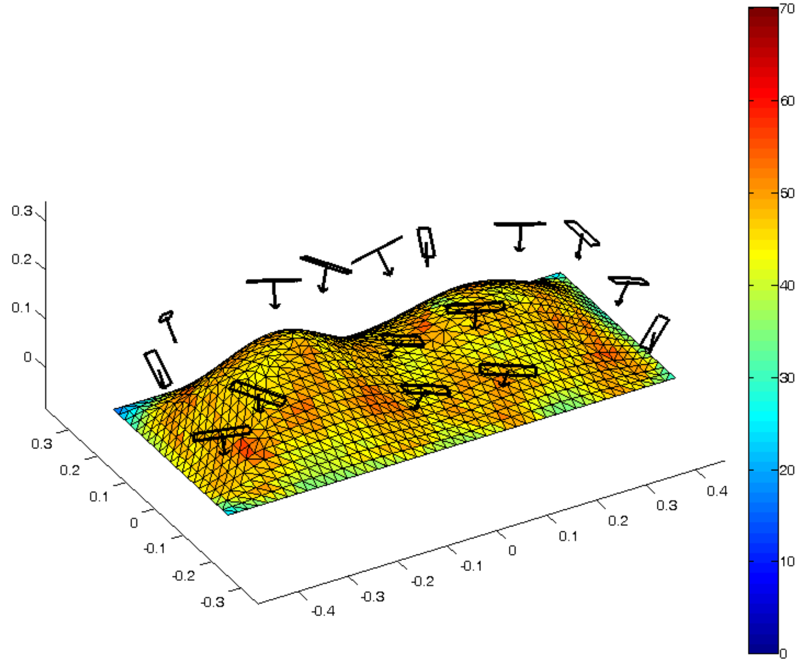

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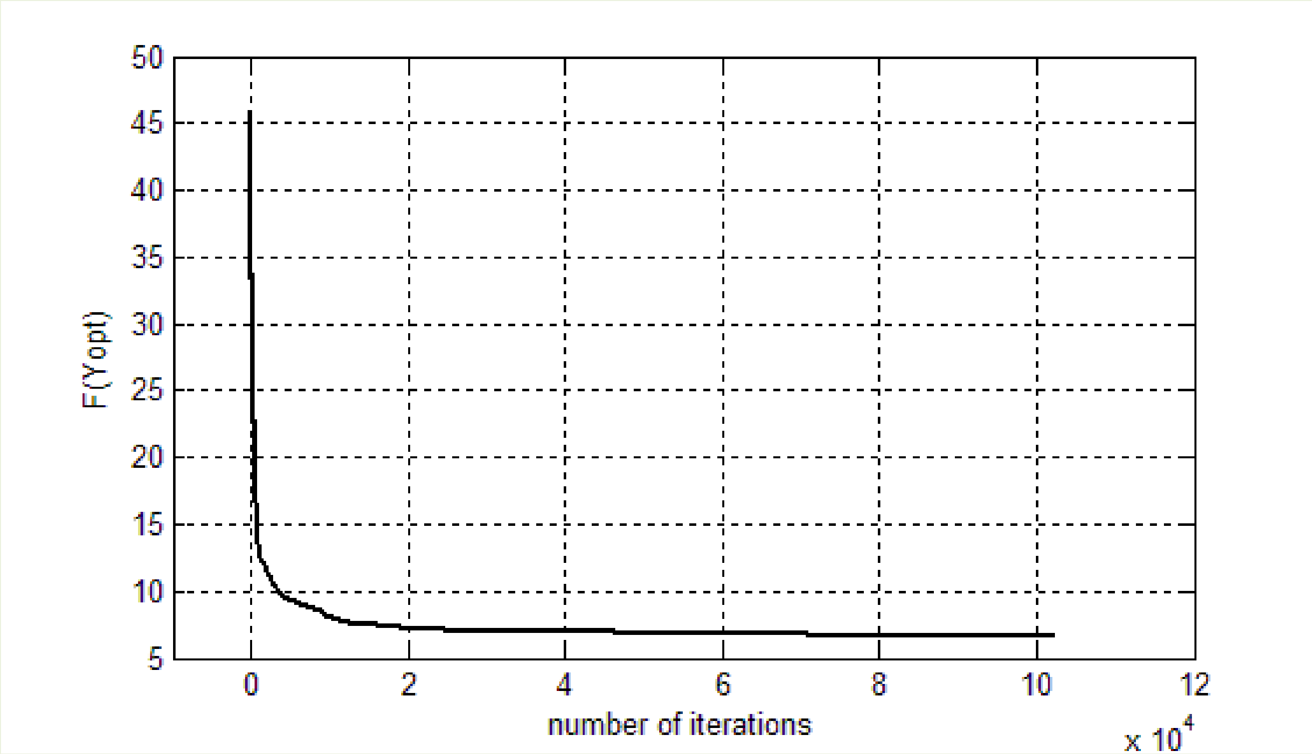
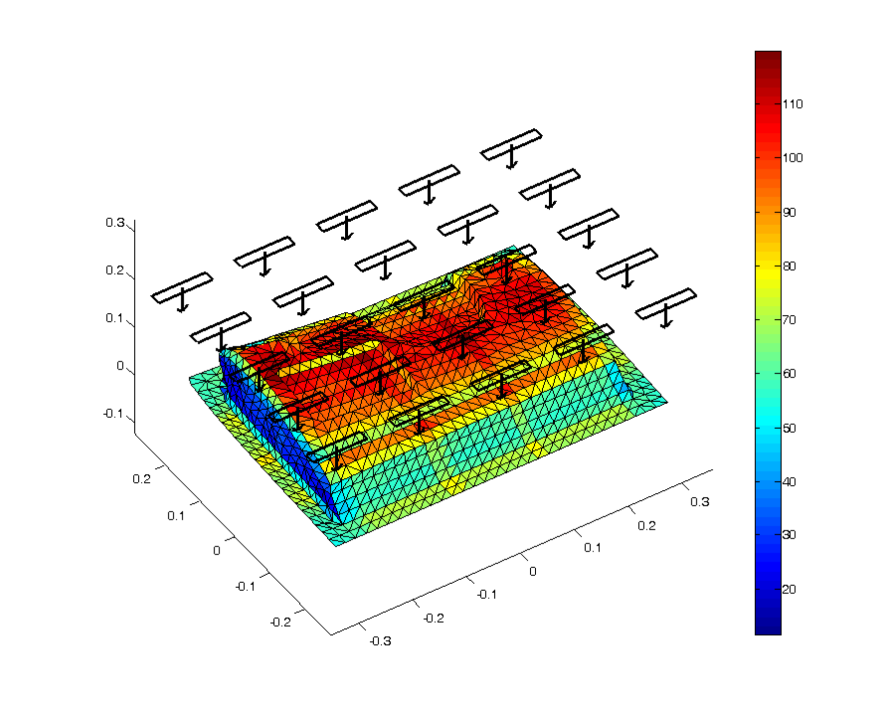
•**Example 3**
- the size of the aluminium mould is 0,6x0,4x0,12[m2]
- 2178 elementary triangular surfaces
- I_{rec}=50[kW/m^2],
- we used 25 heaters
- 100 000 iterations of GA, 5000 iterations of „hill-climbing"

$\tilde{F}(y_1)$=46,52; $\tilde{F}(y_{opt})$=6,56

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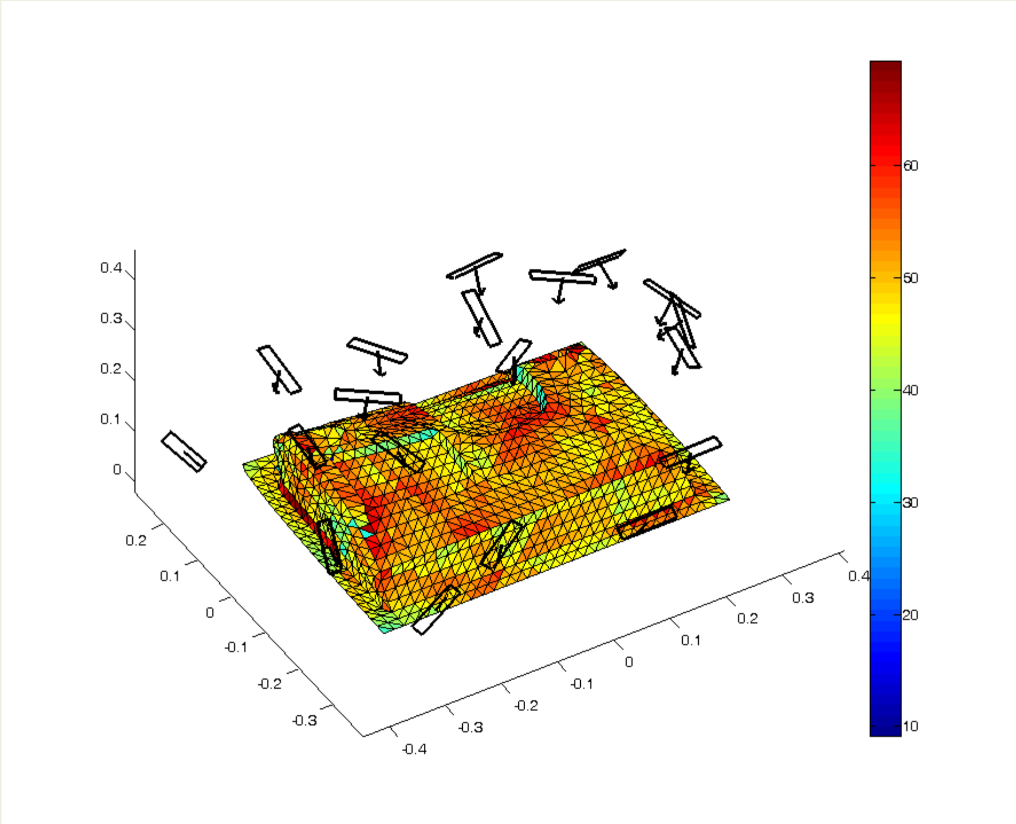
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Thank you for your attention