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ABSTRAKTY

Analytical and numerical algorithms used in solution of inverse problem of the kinematics

Stanislav Bartoň

Abstract: This study deals with an issue of inverse kinematic, that means it facilitates to find a temporal course of vehicle's coordinates a material point in corelation with known behavior of absolute value velocity and osculation circle radius in time. These are kinematic magnitudes that driver himself can affect by choice. Therefore it enables to determine the trajectory and all kinematic magnitudes due to driver's behaviour. The result of this study is a model, which can be used in numerous applications with no other adjustments necessary. That means also a great time and costs saving for developing new applications.

Wavelet bases for the biharmonic problem

Daniela Bímová

Abstract: Wavelets are an established tool for the numerical solution of operator equations. They have namely some interesting properties which may provide an advantage over classical methods if exploited effectively. One of advantages of wavelet methods consists in the existence of a diagonal preconditioner. This preconditioner is optimal in the sense that the condition number of the preconditioned stiffness matrices does not depend on the size of matrices. In our contribution, we study different Riesz wavelet bases in Sobolev spaces based on cubic splines with homogeneous boundary conditions. These bases are consequently applied to the numerical solution of the biharmonic equation and their quantitative properties are compared.

Stručná historie metody konečných objemů

Marek Brandner

Abstrakt: V příspěvku shrneme vývoj metod typu shock capturing pro řešení úloh se zákony zachování. Výklad bude zaměřen na vývoj od klasických diferenčních schémat směrem ke schématům s vysokým rozlišením založeným na metodě konečných objemů. Nejdříve budou popsány nedostatky tradičních diferenčních metod (metoda typu upwind, Laxova-Friedrichsova metoda, Laxova-Wendroffova metoda apod.). Dále budou zmíněny různé metody odstraňující tyto nedostatky: metoda umělé vazkosti, Hartenova-Zwasova metoda, metody typu FCT, UNO, ENO a WENO, Godunovova metoda a některé přibližné Riemannovy řešiče. Stručně okomentujeme některé teoretické poznatky týkající se konzistence, stability a konvergence metod.

Application of analytical solution of Stokes problem in axisymmetric domain to finite element solution of Navier-Stokes equations near corners

Pavel Burda

Abstract: We study the Stokes problem in axisymmetric domain with corners and show the principal part of the analytical solution near corners. The result is applied to finite element solution of the Navier-Stokes equation. Numerical results will be presented.

Volba vah na rozhraní v metodách rozkladu oblasti

Marta Čertíková, Pavel Burda, Jakub Šístek

Abstrakt: Ve svém příspěvku se zabýváme předáváním veličin mezi jednotlivými subdoménami a jejich společným rozhraním u metod rozkladu oblasti bez překryvu subdomén (Substructuring Domain Decomposition Methods). Zejména se soustředíme na metodu BDDC (Balanced Domain Decomposition by Constraints). Zkoumáme různé varianty vážených průměrů a jejich vliv na konvergenci metody.

Averaging of gradient in the space of linear triangular and bilinear rectangular finite elements

Josef Dalík, Václav Valenta

Abstract: We present the averaging method for the second-order approximations of the values of the gradient of an arbitrary smooth function $u = u(x_1, x_2)$ in the vertices of a triangulation T_h composed both of rectangles and triangles in general. The method assumes that the interpolant $P[u]$ of u in the finite element space related to the linear and bilinear finite-elements on the triangles and rectangles from T_h is known only. The second-order approximation of the gradient is applied to the aposteriori error estimates of the errors of the finite-element solutions of the planar elliptic boundary-value problems of second order. Numerical illustrations both of the quality of the averaging method and of the use of the aposteriori error estimates are presented, too.

Perfectly matched layers in computations of coupled mode equations modeling light in photonic crystals

Tomáš Dohnal

Abstract: Wavepackets in periodic structures, like photonic crystals, can be approximated by envelope equations, so called Coupled Mode Equations. These are nonlinear dispersive equations on the whole space where efficient and effective treatment of radiation waves at the edges of the computational domain is crucial. We propose the use of linear Perfectly Matched Layers (PML) under the assumption that only small amplitude waves leave the computational domain. In this approach the domain is surrounded by an artificial layer, where the solution is absorbed while preventing reflections from the interface between the physical and artificial domain. We prove that our PML equations are absorbing and perfectly matched and that they are stable for constant parameters. We also provide numerical experiments to assess the PML performance in both the linear and nonlinear regimes.

Simulation of fluid flow in lower urinary tract

Marek Brandner, Jiří Egermaier, Hana Kopincová

Abstract: We propose a new numerical scheme based on the finite volumes to simulate the urethra flow based on hyperbolic balance law. Our approach is based on the Riemann solver designed for the augmented quasilinear homogeneous formulation. The scheme has general semidiscrete wave–propagation form and can be extended to arbitrary high order accuracy. The first goal is to construct the scheme, which is well balanced, i.e. maintains not only some special steady states but all steady states which can occur. The second goal is to use this scheme as the component of the complex model of the urinary tract including chemical reactions and contraction of the bladder.

Approximate polynomial GCD

Ján Eliaš

Abstract: The computation of polynomial GCD belongs to the basic algebraic problem solved by many authors. In case where polynomials are not given exactly as well as computations are performed in floating point environment, the GCD of given univariate polynomials is calculated only approximately. That is why a term "approximate GCD" (AGCD) is introduced. There are several more or less efficient methods for computing AGCD. Two of them working on Sylvester matrices are compared. Both methods are completely new in the field of AGCD computation. The first one transforms the stated problem into the minimization problem, i.e. AGCD is found as a minimum of some function. Advantage of this method is that various optimization techniques can be used. On the other side, the second discussed structure preserving method works with the useful structure of the Sylvester matrix. Numerical examples are presented.

Discontinuous Galerkin techniques for the interaction of compressible flow and structures

Miloslav Feistauer

Abstract: The paper will be concerned with the numerical solution of compressible flow in time dependent domains with applications to fluid-structure interaction. The motion of the boundary of the domain occupied by the fluid is taken into account with the aid of the ALE (Arbitrary Lagrangian-Eulerian) formulation of the Euler and Navier-Stokes equations describing compressible flow. They are discretized by the discontinuous Galerkin finite element method (DGFEM) using piecewise polynomial discontinuous approximations. For the time discretization two techniques are applied. The first possibility is a semi-implicit linearized BDF scheme with extrapolation in nonlinearities. Second technique, which is more robust, is the time discontinuous Galerkin linearized method. In both cases, only one linear algebraic system is solved on each time level. Important ingredients are suitable treatment of boundary conditions and shock capturing based on the discontinuity and boundary/internal layers indicator and local artificial viscosity. As a result we get efficient and robust numerical processes allowing the solution of compressible flow with a wide range of Mach and Reynolds numbers. The applicability of the developed method will be demonstrated on several problems.

A quadratic wavelet basis on the interval

Václav Finěk

Abstract: In signal and image processing as well as in numerical solution of differential equations, wavelets with short support and high order of vanishing moments are of interest because they have good approximation properties and provide fast algorithms. A B-spline of order m is a spline function that has minimal support among all compactly supported refinable functions with respect to a given smoothness. And recently, B. Han and Z. Shen constructed a Riesz wavelet bases of $L_2(\mathbb{R})$ with m vanishing moments based on B-spline of order m. In our contribution, we present an adaptation of quadratic wavelets to the interval $[0, 1]$ which preserves vanishing moments.

Massive parallel implementation of ODE solvers.

Cyril Fischer

Abstract: The proposed contribution maps the possibilities of exploitation of the massive parallel computational hardware (namely GPU) for solution of the ordinary differential equations. Two cases are discussed: parallel solution of the single ODE and parallel execution of scalar ODE solvers. Whereas the advantages of the special architecture in the case of single ODE are problematic, repeated solution of a single ODE for different data can profit from the parallel architecture. However, special algorithms have to be used even in the latter case to avoid code divergence between individual computational threads. The topic is illustrated on the example of response spectra computation.

Adaptive finite element method for second order formulations of neutron transport

Milan Hanuš

Abstract: In this talk, we will show how the first-order neutron transport equation can be transformed into a second-order PDE (or system of such equations) in various ways, including the simple multigroup diffusion approximation and the more advanced simplified PN approximation. Then, we will present a general framework for solving these equations, built upon the hp-FEM library Hermes. Use of this framework will be demonstrated on several neutron transport problems and performance of various h- and hp-adaptivity options available in the library will be discussed. Special tuning of these adaptivity techniques for the aforementioned neutron transport approximations will also be covered.

An extension of small-strain models to the large-strain range based on an additive decomposition of a logarithmic strain

Martin Horák

Abstract: In this paper we will explore an extension of small-strain models to the large strain range. The extension to the large-strain range is based on the additive decomposition of the logarithmic strain into elastic and plastic parts. The main advantage of this approach is its modular structure, consisting of three steps:

1. Definition of the elastic and plastic parts of the logarithmic strain.
2. Computation of the generalized stress tensor, work-conjugate to the logarithmic strain, and of the appropriate generalized stiffness, via an algorithm that preserves the structure of the small-strain theory.
3. Transformation of the generalized tensors to the second Piola-Kirchhoff stress and the corresponding material stiffness.

An efficient implementation into an object-oriented finite element platform will be presented, the algorithmic aspects will be discussed and application of the model will be illustrated by an example.

Valuing barrier options using the adaptive discontinuous Galerkin method

Jiří Hozman

Abstract: The valuation of different types of option contracts is very important in modern financial theory and practice. Most of the analytical formulas for these options is limited with strong assumptions, which led to apply numerical methods instead. This paper is devoted to the barrier options and the main objective is to develop sufficiently robust, accurate and efficient method for computation of their values driven according to the well-known Black-Scholes equation. The main idea is based on the discontinuous Galerkin method together with a spatial adaptive approach. This combination seems to be a promising technique for the solving of such problems with discontinuous solutions as well as for optimization of the number of degrees of freedom and computational demandingness, consequently. A set of numerical experiments illustrating the potency of the proposed scheme is presented.

Úloha optimalizace parametrů *hp* verze metody konečných prvků

Jan Chleboun

Abstrakt: Cílem úlohy je při daném počtu stupňů volnosti optimalizovat jak síť, tak stupně polynomů tvořících bázi aproximačního prostoru, přičemž má být dosaženo minimálního rozdílu mezi přesným a přibližným řešením 1D eliptického okrajového problému.

Integro-differential equations with time-varying delay

Pavol Chocholatý

Abstract: Integro-differential equations with time-varying delay (IDETVD) can provide us with realistic models of many real word phenomena. Delayed Lotka - Volterra predator-prey systems arise in Ecology. We investigate the numerical solution of a linear system of two IDETVD and the given initial function. We will present an approach based on k-step methods using quadrature formulas. Numerical solution can be controlled by monitoring the size of the defect of the approximate solution and adjusting the step size on each step of the integration. Numerical results will be presented to demonstrate the effectiveness of this approach.

Numerical study of Weisenberg number for Newtonian and Oldroyd-B fluids flow

Radka Keslerová, Karel Kozel

Abstract: This work deals with the numerical solution of laminar incompressible viscous and viscoelastic flow in 2D channel for Newtonian and Oldroyd-B fluids. The governing system of equations is based on the system of balance laws for mass and momentum. Numerical solution of the described models is based on cell-centered finite volume method using explicit Runge-Kutta time integration. The flow is modelled in a bounded computational domain. Numerical results obtained by this method are presented and compared.

Tangential fields in the optical diffraction problems

Jiří Krček, Jaroslav Vlček, Arnošt Žídek

Abstract: Optical diffraction for periodical interface belongs to relatively fewer exploited application of boundary integral equations (BIE) method. Our contribution presents the formulation of diffraction problem based on vector tangential fields, for which the periodical Green function of Helmholtz equation is of key importance. There are discussed properties of obtained boundary operators with singular kernel, and, a numerical implementation is proposed.

Řešení sdruženého modelu transportu tepla a podzemní vody v pórovitém prostředí

Lukáš Krupička

Abstrakt: V příspěvku se budeme zabývat odvozením a numerickým řešením modelu sdruženého transportu tepla a podzemní vody v nehomogenním pórovitém prostředí. Matematický model je popsán systémem dvou evolučních nelineárních diferenciálních rovnic (Richardsova rovnice a rovnice vedení tepla) s odpovídajícími počátečními a okrajovými podmínkami. Metoda časové diskretizace vede na problém řešení soustavy nelineárních okrajových úloh s neznámým rozložením teploty a tlakové výšky. Pro uvedený problém dokážeme existenci řešení a uvedeme numerické výsledky rozložení teploty a režimu proudění vody v půdním profilu.

Error estimates for nonlinear convective problems in finite element methods

Václav Kučera

Abstract: Standard apriori error estimates for finite element methods use tools which are suitable for the treatment of diffusive or diffusion-dominated equations. One uses the nice structure of these terms (e.g. ellipticity or monotonicity) to obtain error estimates by Gronwall's lemma or similar standard tools. This so-called parabolic technique is suitable for convection-diffusion problems (e.g. Navier-Stokes) only if the convective terms, which have no known ”nice structure”, can be absorbed by the diffusion terms. This leads to estimates which blow up exponentially with the diffusion coefficient going to zero and which are not valid in the purely convective case. In this work, we shall present error estimates for nonlinear purely convective problems essentially using the parabolic technique. We build on results of Zhang, Shu (2004) originally dealing with explicit discontinuous Galerkin schemes. We extend their results, which rely heavily on mathematical induction, to the method o f lines via so-called continuous mathematical induction and a non-linear Gronwall lemma. For implicit schemes, we show that standard arguments cannot prove the desired estimates. To circumvent this obstacle, we construct a suitable continuation of the discrete solution with respect to time, so that we can again apply continuous mathematical induction. For simplicity of presentation, we shall consider standard finite elements, although the results can be extended to the discontinuous Galerkin method.

Numerické modelování fyzikálně-mechanických stavů proudících kapalin v reálném prostředí technického prvku

Vojtěch Kumbár

Abstrakt: Tento článek pojednává o numerickém modelování fyzikálně-mechanických stavů proudících kapalin v reálném prostředí technického prvku. Konkrétně se jedná o stanovení tlakových a rychlostních poměrů podél geometrie modelu ve zvolených místech pro tři různé teplotně závislé materiály (motorové oleje různých viskozitních tříd) při třech různých teplotách proudícího média (0 °C, 20 °C a 70 °C). Jsou sledována především místa za zakřivením geometrie (celkem 6 pozic).

Multi-criteria optimization of induction heating problem

František Mach, <u>Pavel Kůs</u>, Pavel Karban, Ivo Doležel

Abstract: We present an example of multi-criteria optimization in the finite element simulations. The multi-criteria optimization is used in engineering design in situations where more criteria are optimized, e.g. when we want to find parameters defining shape of a device to maximize power (first criterion) and minimize consumption (second criterion). Optimization algorithms should provide a set of possible trade-offs (the so called Pareto front), from which one particular design can be selected. The algorithm will be showed for an induction heating optimization problem. Finite element calculations are performed using the Agros2D system. The optimization algorithm has to use as few criteria evaluations as possible, since they require solution of a direct problem and hence might be very expensive.

Optimalizace osvitu pro tepelný ohřev forem v automobilovém průmyslu

<u>Ladislav Lukšan</u>, Jiřina Královcová, Jaroslav Mlýnek

Abstrakt: Je navržen model osvitu hliníkových forem používaných v automobilovém průmyslu. Tento model je analyzován a jsou odvozeny jeho citlivosti na změnu parametrů zahřívacích lamp, které slouží k výpočtu gradientu účelové funkce. Účelová funkce má tvar součtu čtverců a je třeba ji minimalizovat na množině zadané nelineárními omezeními ve tvaru rovností. Je popsána optimalizační metoda sloužící k optimálnímu nastavení parametrů zahřívacích lamp, jsou uvedeny procedury realizující účelovou funkci a omezující funkce a jsou ukázány výsledky získané aplikací tohoto postupu v případě modelových úloh.

On estimation of diffusion coefficient based on spatio-temporal FRAP images: An inverse ill-posed problem

Ctirad Matonoha, Štěpán Papáček, Jindřich Soukup

Abstract: This study describes the method of estimation of diffusivity (diffusion coefficient D) of a fluorescent particle in an Euclidean bounded domain based on spatio-temporal FRAP (Fluorescence Recovery After Photobleaching) images. The FRAP technique is rutinary used to the investigation of protein dynamics within the living cells, in our case we aim to study the mobility of photosynthetic complexes in a native intact membrane. The diffusion process is modelled by the Fickian diffusion PDE with the noisy initial condition and time-varying (experimentally measured) Neumann boundary conditions. The single parameter estimation problem is then formulated as the optimization problem residing in the minimization of an objective function representing the disparity between the experimental and simulated data. Our inverse problem is ill-posed thus we have to use a regularization technique. Two regularization algorithms, with one parameter and two parameters, were implemented. On the benchmark problem, the numerical experiments show better performance of the two-parameter regularization algorithm.

Optimalizace intenzity tepelného záření na povrchu formy

Jaroslav Mlýnek, Radek Srb

Abstrakt: Článek je zaměřen na problematiku zahřívání povrchu hliníkové formy pomocí infrazářičů při výrobě umělých kůží v automobilovém průmyslu. Cílem je nalézt vhodné umístění infrazářičů nad formou, které zajistí intenzitu tepelného záření rovnoměrnou na celém povrchu formy a blížící se hodnotě doporučené výrobcem. V úvodu článku je popsán matematický model ozařování formy, povrch formy je zadán pomocí jednotlivých elementárních ploch. Výpočet intenzity záření v okolí infrazářiče je prováděn interpolací experimentálně naměřených hodnot intenzity záření ve vybraných bodech. Optimalizace umístění infrazářičů nad formou je realizována užitím genetického algoritmu. V průběhu výpočtu je testováno, zda-li nedochází ke kolizím (mezi zářiči a povrchem formy a zářiči). Úloha byla naprogramována v jazyku Matlab. V závěrečné části článku je uveden praktický příklad optimalizace nastavení infrazářičů nad formou. Součástí uvedeného řešení je i grafická prezentace výsledků.

Shape functions and wavelets - tools of numerical approximation

Vratislava Mošová

Abstract: Solution of boundary value problem is often realized as the application of the Galerkin method to the weak formulation of this problem. The choice of the approximating finite dimensional subspace is important matter. It is possible to generate this subspace by means of splines or by means of functions that are not polynomial and have compact support. We restrict our attention only to RKP-shape functions and wavelets in the contribution. Common features and comparision of approximation properties of these functions will be studied.

The total least squares problem with multiple right-hand sides

Iveta Hnětynková, Martin Plešinger, Diana M. Sima, Zdeněk Strakoš, and Sabine Van Huffel

Abstract: The *total least squares (TLS) techniques*, also called *orthogonal regression* and *errors-in-variables modeling*, have been developed independently in several areas. For a given linear (orthogonally invariant) approximation problem $AX \approx B$, where $A \in \mathbb{R}^{m\times n}$, $B \in \mathbb{R}^{m\times d}$, $X \in \mathbb{R}^{n\times d}$, the TLS formulation aims at a solution of a modified problem

$$(A+E)X = B+G \qquad \text{such that} \qquad \min \|[G,E]\|_F .$$

The algebraic TLS formulation has been investigated for decades ([2], [1, Section 6], [5]). In [3] it is shown that even with $d = 1$ (which represents a problem with the *single right-hand side* $Ax \approx b$, where b is an m-vector) the TLS problem may not have a solution and when the solution exists, it may not be unique. The classical book [6] introduces the *generic-nongeneric* terminology representing a commonly used classification of TLS problems. If $d = 1$, then the *generic problems* represent problems that have a (possibly nonunique) solution, whereas *nongeneric problems* do not have a solution. This is no longer true for $d > 1$. For $d > 1$, [6] analyzes only two representative cases characterized by the special distribution of singular values of the extended matrix $[B, A]$. A general case is not analyzed—it is considered only as a perturbation of one of the special cases. The so called *classical TLS algorithm* in [6] however computes some output X for any data A, B. The relationship of the output X to the original problem is not clear.

In the first part of our contribution we try to fill this gap and investigate existence and uniqueness of the solution of the TLS problem with $d > 1$ in full generality. We suggest a classification of TLS problems revisiting and refining the basic generic-nongeneric terminology. A core reduction concept introduced in [4] makes a clear link between the original data and the output of the classical TLS algorithm for the problems with $d = 1$. In the second part of this contribution we introduce an extension of the core reduction for multiple right-hand sides problems. Using the core reduction, we illustrate some particular difficulties which are present in the TLS problems with $d > 1$.

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An optimal algorithm with Barzilai-Borwein steplength and superrelaxation for QPQC problem

Lukáš Pospíšil

Abstract: We propose a modification of MPGP algorithm for solving minimizing problem of strictly convex quadratic function subject to separable spherical constraints. This active set based algorithm explores the faces by the conjugate gradients and changes the active sets and active variables by the gradient projection with the Barzilai-Borwein steplength. We show how to use the algorithm for the solution of separable and equality constraints. The power of our modification is demonstrated on the solution of a contact problem with Tresca friction.

Numerical solution of viscoelastic flows

Vladimír Prokop

Abstract: This work concerns with numerical simulation of viscoelastic flows based on the Oldroyd-B model. Newtonian model of a fluid is not able to capture all the phenomena in many fluids with complex microstructure, where are involved much larger scales than the atomic scale. The motion of viscoelastic fluids is described by the conservation of mass and momentum. The mathematical model is complete when a constitutive law, relating extra stress tensor to the motion, is prescribed. The extra stress tensor depend not only on the current motion of the fluid, but also on the history of the motion. Finite volume and Runge-Kutta methods are used to solve the resulting system of equations.

Běžný grafický hardware jako superpočítač

Jan Přikryl

Abstrakt: S rozmachem výkonných grafických karet přišli jejich tvůrci s ideou doplnit proprietární hardware programovacím rozhraním, jež umožňuje použít výpočetní výkon grafických procesorů (GPU) i pro jiné úlohy, než je vykreslování složitých 3D scén. Vznikly tak zvané GPGPU (General-purpose GPU) procesory, jež - pokud jsou správně naprogramovány - dosahují zcela ohromujících poměrů cena/výkon v operacích v pohyblivé řádové čárce. V příspěvku se budu věnovat technologii nVIDIA CUDA a budu demonstrovat proces vývoje GPGPU aplikace jak v nativním kódu GPU, tak pomocí nadstaveb pro výpočetní balík MATLAB (Jacket, Parallel Toolbox).

Algebraický multigrid pro stochastické matice

Ivana Pultarová

Abstrakt: Víceúrovňové metody jsou populárním prostředkem pro řešení soustav lineárních rovnic, zejména těch, které vzniknou diskretizací diferenciálních rovnic. Mezi takové metody patří metody algebraického multigridu. Jejich varianta pro řešení úloh s obecně nesymetrickými stochastickými maticemi se nazývá iterační agregační - desagregační metody. Konvergence těchto metod však není pro nesymetrické matice zaručena a to ani v lokálním smyslu. V příspěvku ukážeme nový výsledek: vzorec pro chybu přibližného řešení získaného iterační agregační - desagregační metodou s libovolným počtem úrovní. Použijeme tento vzorec pro předvedení některých překvapivých vlastností této metody.

Topological degree computation and applications

Stefan Ratschan, Peter Franek

Abstract: We present a new algorithm for computing the topological degree of analytic n-dimensional real functions given in the form of arithmetical expressions. The algorithm cleanly separates numerical from combinatorial computation. Based on this, the numerical part provably computes only the information that is strictly necessary for the following combinatorial part, and the combinatorial part may optimize its computation based on the numerical information computed before. We also report on computational experiments and applications in the verification of zeros of systems of equalities, and in computational logic.

Dva přístupy k vynucení teploty při chlazení proudící vodou

Petr Salač

Abstrakt: Lokální změnu teploty na pracovním povrchu lisovacího nástroje můžeme docílit buď změnou tepelného odporu chladícího nástroje, již realizujeme změnou tloušťky stěny, anebo lokální změnou intenzity chlazení na „správném“ místě chladící dutiny, již realizujeme lokální změnou rychlosti proudící chladící tekutiny. Citlivostní analýza založená na homeomorfizmu tepelných toků definuje k danému místu pracovního povrchu „správné“ místo návrhového povrchu, jež ovlivňuje změnu lokální teploty na pracovním povrchu. Velikost změny v místě návrhu je potom přímo úměrná přírůstku velikosti energie potřebné k dosažení požadované teploty. V příspěvku jsou představeny dvě úlohy tvarové optimalizace určené k vyrovnání povrchové teploty razníku založené na výše uvedených principech.

Smooth approximation of data with applications to interpolating and smoothing

Karel Segeth

Abstract: In the contribution, we are concerned with some computational aspects of smooth approximation of data. This approach to approximation employs a (possibly infinite) linear combination of smooth functions with coefficients obtained as the solution of a variational problem, where constraints represent the conditions of interpolation or smoothing. Some 1D numerical examples are presented.

On comparison of different stabilizations of finite element methods in aeroelastic problems

Petr Sváček

Abstract: This paper focuses on the mathematical and numerical modelling of interaction of the two-dimensional incompressible fluid flow and a flexibly supported airfoil section with control section. A simplified problem is considered: The flow is modelled by the system of Navier-Stokes equations and the structure motion is described with the aid of nonlinear ordinary differential equations. The time-dependent computational domain is taken into account by the Arbitrary Lagrangian-Eulerian method. Higher order time discretization is considered within stabilized finite element methods. Several types of stabilizations are considered and the application is shown.

Efektivní násobení s waveletovou maticí

Václav Finěk, Martina Šimůnková

Abstrakt: Řešíme-li Dirichletovu okrajovou úlohu

$$-u'' + cu = f$$

s homogenní okrajovou podmínkou Galerkinovou metodou a zvolíme-li za bázi splinové wavelety, má dle [1] matice tuhosti $O(n \log n)$ nenulových prvků. Ukážeme, že nenulových prvků je $O(n)$ a předvedeme algoritmus, který počítá součin této matice s vektorem v $O(n)$ operacích. Výše uvedenou matici včetně předvedeného algoritmu násobení použijeme při řešení vícerozměrné úlohy

$$-\sum_{i=1}^{d} \frac{\partial^2 u}{\partial x_i^2} + cu = f$$

na hyperkrychli $[0, 1]^d$.

Literatura:

[1] Urban, Karsten: Wavelet Methods for Elliptic Partial Differential Equations, Oxford University Press, 2009.

Primární metody iterativního dělení na podoblasti

Jakub Šístek

Abstract: V první polovině přednášky poskytneme přehled nejčastěji užívaných metod rozkladu na podoblasti (domain decomposition). Popíšeme princip iterativního dělení na podoblasti (iterative substructuring) a jak na něj metody domain decomposition navazují. Vysvětlíme rozdíly mezi primárními a duálními metodami, mezi metodami s překryvem a bez překryvu. V přehledu postihneme chronologický vývoj od metody Neumann-Neumann, přes BDD a FETI, až po FETI-DP a BDDC. Právě metodě BDDC (Balancing Domain Decomposition by Constraints) pak bude věnována druhá část přednášky, ve které bude detailně vysvětlen její algoritmus a výhody. Zmíníme se o víceúrovňové a adaptivní verzi metody BDDC. V závěru budou na metodě BDDC diskutovány některé aspekty paralelní implementace metod rozkladu na podoblasti a ukázky aplikací.

Adaptive higher-order finite element methods for transient PDE problems

Pavel Šolín

Abstract: We present a new class of adaptivity algorithms for time-dependent partial differential equations (PDE) that combine adaptive higher-order finite elements (*hp*-FEM) in space with arbitrary (embedded, higher-order, implicit) Runge-Kutta methods in time. Weak formulation is only created for the stationary residual, and the Runge-Kutta methods are specified via their Butcher's tables. Around 30 Butcher's tables for various Runge-Kutta methods with numerically verified orders of local and global truncation errors are provided. A time-dependent benchmark problem with known exact solution that contains a sharp moving front is introduced, and it is used to compare the quality of seven embedded implicit higher-order Runge-Kutta methods. Numerical experiments also include a comparison of adaptive low-order FEM and *hp*-FEM with dynamically changing meshes.

Networked computing laboratory (NCLab)

Pavel Šolín

Abstract: NCLab is a web framework for programming, mathematics, computer modeling and scientific computing. Its objectives differ from commercial software packages such as Matlab, Maple, MathCAD, Comsol, Ansys and others. It provides a mechanism for researchers to develop interactive graphical applications based on their own computational methods, and make them instantly available to vast amounts of users. NCLab is powered by cloud computers and it works entirely in the web browser window. It uses advanced networking technologies to provide a highly creative atmosphere of sharing and real-time collaboration. Users can access their accounts on anytime-anywhere basis, including from mobile devices, meet in NCLab and work there together. The only requirement is a fast Internet connection. The framework is still in development, but it already has around 2500 regular users. In this presentation we will describe basic features of NCLab and focus on modules for geometrical modeling, mesh generation, and postprocessing that can be attached to any finite element code that complies with their simple APIs. Time permitting we will show other things that one can do in NCLab such as Solid Modeling using the PLaSM library and WebGL, and GPU computing with CUDA.

On computing quadrature-based bounds for the A-norm of the error in conjugate gradients

Petr Tichý

Abstract: In their original paper, Golub and Meurant suggest to compute bounds for the A-norm of the error in the conjugate gradient (CG) method using various types of quadratures. The quadratures are computed using the $(1,1)$-entry of the inverse of the corresponding Jacobi matrix (or its rank-one or rank-two modifications). The resulting algorithm called CGQL computes explicitly the entries of the Jacobi matrix and its modifications from the CG coefficients. In this contribution, we use the fact that CG computes the Cholesky decomposition of the Jacobi matrix which is given implicitly. For Gauss-Radau quadrature, instead of computing the entries of the modified Jacobi matrices, we directly compute the entries of their Cholesky decompositions. This leads to simpler formulas in comparison to those used in CGQL.

Numerické aspekty identifikace tepelných charakteristik metodou topného drátu

Jiří Vala

Abstrakt: Metoda topného drátu, založená na měření vývoje teploty ve zkušebním vzorku, v němž je zavedena sonda známých vlastností s integrovaným tepelným zdrojem, umožňuje stanovení tepelné vodivosti materiálů i při extrémních namáháních, např. u žáruvzdorných vyzdívek. Vzorce uvedené v technické normě vycházejí z analytického řešení nestacionární rovnice vedení tepla v cylindrických (zjednodušeně polárních) souřadnicích pro neadekvátní počáteční a okrajové podmínky, což zpochybňuje validitu výsledků a brání současnému stanovení tepelné kapacity. Numerická analýza pro korektní podmínky metodou konečných diferencí, prvků, objemů apod. je problematická kvůli řádové odlišnosti tloušťky drátu a rozměrů vzorku. Příspěvek ukazuje možnost řešení naznačené inverzní úlohy, jejímž cílem je určení tepelné vodivosti i kapacity, s využitím vlastností Besselových funkcí a metody nejmenších čtverců. Navržený algoritmus je implementován v prostředí MATLABu.

Guaranteed estimates of the constant in Friedrichs' inequality

Tomáš Vejchodský

Abstract: We present an approach for computation of guaranteed upper bounds of the optimal constant in Friedrichs' and similar inequalities. The approach is based on the method of a priori a posteriori inequalities of Kuttler and Sigillito. The original method requires trial and test functions with continuous second derivatives. We show how to avoid this requirement and how to compute the bounds on Friedrichs' constant by using standard finite element methods. This approach is quite general and allows variable coefficients and mixed boundary conditions. We use the computed upper bound on Friedrichs' constant in a posteriori error estimation to obtain guaranteed error bounds.

A priori diffusion–uniform error estimates for singularly perturbed problems – DG and higher order time discretizations

Miloslav Vlasák, Václav Kučera

Abstract: We deal with a nonstationary semilinear singularly perturbed problem. A priori error analysis for singularly perturbed problems represents a very difficult task and a majority of the results concerned with this topic include numerical analysis for the linear case only. We will focus on the numerical analysis of our semilinear problem discretized by the discontinuous Galerkin method (DG) in space and by several higher order schemes in time and we will derive diffusion uniform estimates for these discretizations.

Modifikace metody BFGS s omezenou pamětí, založené na myšlence sdružených směrů

Jan Vlček, Ladislav Lukšan

Abstrakt: Jsou vyšetřovány jednoduché modifikace metody BFGS s omezenou pamětí (L-BFGS) pro nepodmíněnou minimalizaci, spočívající ve vektorových korekcích, využívajících hodnot z předchozí iterace a odvozených z myšlenky sdružených směrů. Pro kvadratické účelové funkce je zlepšení konvergence nejlepší možné v jistém smyslu a všechny uložené směrové vektory jsou vzájemně sdružené pro jednotkové délky kroku. Navržený algoritmus je globálně konvergentní a numerické výsledky naznačují, že zlepšení efektivity je významné.

FFT-based finite element method for homogenization of periodic media

Jan Zeman

Abstract: In this talk, I will overview our recent results related to a numerical algorithm for the solution of the periodic unit cell problem proposed by Moulinec and Suquet in 1994. In its original variant, the method is based on a problem reformulation in the form of a Lippmann-Schwinger equation solved by the Neumann series expansion efficiently combined with the Fast Fourier Transform (FFT). We show that, quite surprisingly, this setting is equivalent to a finite element discretization with trigonometric polynomials as basis function and with FFT method used to perform the numerical integration. Moreover, we demonstrate that the resulting non-symmetric system of linear equations is solvable by standard conjugate gradient algorithm, which leads to substantial improvements over the original Moulinec-Suquet scheme. This is a joint work with Jaroslav Vondrejc and Ivo Marek (CTU in Prague).

Calculation of the greatest common divisors of perturbed polynomials

Jan Zítko, Ján Eliaš

Abstract: The coefficients of the greatest common divisor of two polynomials f and g are usually calculated from the Sylvester matrix $S(f,g)$. If inexact polynomials are given then the approximate greatest common divisor is calculated and a process called a structured low rank approximation is considered. The STLN algorithm is practical implementation of a just mentioned process. The second technique is based on the rank revealing algorithm which is used for finding the first rank deficient Sylvester subresultant matrix.
All just mentioned techniques are described, discussed and numerically compared in the presented lecture.