Decompositions of isometries on Hilbert spaces defined by measure decompositions.

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Lebesgue decomposition of a measure generates a decomposition of an operator. Precisely for an isometry (contraction) on a Hilbert space $V \in \mathcal{B}(\mathcal{H})$ and a vector $x \in \mathcal{H}$ the mapping $\mu_x : \mathbb{B}(\mathbb{T}) \ni \omega \to \langle E(\omega)x, x \rangle$ is a positive Borel measure where E stands for a spectral measure of the minimal unitary extension (dilation) of V. The measure μ_x is called an *elementary measure* of x and V. Lebesgue decomposition of an isometry is a decomposition into an absolutely continuous isometry and a singular isometry which are isometries having all elementary measures of respective type.
We consider another decomposition of a measure - Szegö decomposition. A Borel measure μ on the σ-algebra of all Borel subsets of the unit circle $\mathbb{B}(\mathbb{T})$ is called a *Szegö measure*, if for any $\omega \in \mathbb{B}(\mathbb{T})$ the inclusion $\chi_\omega L^2(\mu) \subset H^2(\mu)$ implies $\mu(\omega) = 0$. The measure μ is *Szegö singular* if $H^2(\mu) = L^2(\mu)$. An arbitrary Borel measure can be decomposed into a sum of a Szegö measure and a Szegö singular measure. The generalization of Szegö decomposition to operators is not as direct as in the case of Lebesgue decomposition. Since linear combination of vectors which elementary measures are Szegö may be any type, even Szegö singular then an isometry $V \in \mathcal{B}(\mathcal{H})$ is defined to be *Szegö isometry* if $\mathcal{H}$ is spanned by vectors which elementary measures are Szegö. An isometry is called *Szegö singular* if an elementary measure of any vector is Szegö singular. An isometry can be decomposed into Szegö isometry and Szegö singular isometry. However, such a decomposition in case of operators turns out to be equivocal. Consequently there can be defined a family of such decompositions called Szegö type decompositions. Two such decompositions be presented. Some of their properties and applications be discussed. The idea can be extended to contractions via unitary dilations.