

A GENERAL APPROACH TO THE CONCEPT OF SECOND DUAL SPACES

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We develop a concept, that generalizes the idea of the second dual space of a normed vector space in a fairly general way. The main tool is to recognize the "first dual" as a means to the end of the second dual:

In order to apply the same process of dualization twice, it seems to be necessary, to transfer the entire kind of structure of the base space X to its dual X^d . Unfortunately, this is impossible in much other cases than normed vector spaces - because the algebraic structure hardly struggles.

However, the most interesting results in our perception here, are concerned with the *second* dual X^{dd} . So, in this context we propose to relinquish the idea of *one concept* of duality that has to be applied *twice* to bring out a powerful tool. Instead, we describe a fairly general procedure to get very appropriate second duals X^{dd} using just such "first duals", whose construction is no longer solicitous to reproduce the structure of the original space X in the function space X^d but to enable such a structure in the function space X^{dd} with the same range, but with domain X^d .

Especially, we can easily prove some essential statements on embeddings of noncommutative C^* -algebras in their second dual, as whose analogues are known in the commutative setting.

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