Non-separable Banach spaces studied via "rich" families of their separable subspaces

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Interactions between Algebra and Functional Analysis

Prague, 28 September - 2 October 2016

We present a structural statement characterizing Asplund space X via a certain "rich" family of separable "rectangles" $V \times Y \subset X \times X^*$. This enables to construct easily a "projectional skeleton" (a modern substitute of PRI) in X^*. Further we find some rich "rectangle" families in $X \times X^*$ where X is WCG, ..., WLD, or even Pličko space. Thus we get a "commutative" projectional skeleton in X. The class of Pličko spaces is quite large. It contains $L_1(\mu)$, with μ σ-finite, order continuous lattices, $C(G)$, with G a compact abelian group, and preduals of semifinite von Neumann algebras. Finally, if X is simultaneously Asplund and WCG, the previous results put together immediately yield a commutative projectional skeleton in X so that the adjoint projections form a projectional skeleton in X^*. The lecture is based on three papers written jointly with Marek Cúth.