

Exercises for Mathematical Logic (21 Dec 2022)

In the next three exercises, you will develop an alternative sequence encoding scheme due to Edward Nelson.

23. The set $\{x : \exists n \in \mathbb{N} x = 2^n\}$ of powers of 2 is definable by a Δ_0 formula, not using the 2^n function. [Hint: Consider the divisors of x .]

24. Consider an encoding of finite sets $X \subseteq \mathbb{N}$ by pairs $\langle r, w \rangle$ where the binary expansion of r acts as a “ruler” with marks at positions of 1s, and the binary expansion of w is a concatenation of binary expansions of elements of X such that each element occupies the position between two ruler marks. Show that the predicate “ x is in the set encoded by $\langle r, w \rangle$ ” is Δ_0 -definable.

25. Construct a Δ_0 encoding of finite sequences based on the previous exercise.

A function $f: \mathbb{N} \rightarrow \mathbb{N}$ is *represented* by a formula $\varphi(x, y)$ in a theory T if $T \vdash \forall y (\varphi(\bar{n}, y) \leftrightarrow y = \bar{m})$ for all $n, m \in \mathbb{N}$ such that $f(n) = m$.

26. All partial computable functions are representable in \mathcal{Q} . [Hint: Using Σ_1 -definability of the graph of f , adapt the witness comparison argument from the proof of representability of decidable sets.]